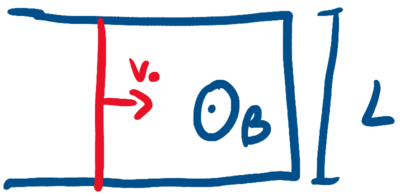
.Appello 26/05/2022 (Autovalutazione)

lunedì 15 gennaio 2024 11:04

Es 1



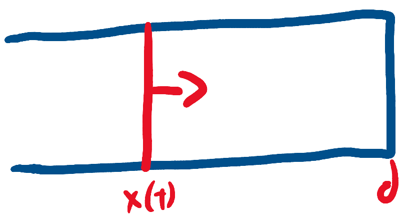
$m = 0,5\ kg$

$B = 1T$

$R = 10\ \Omega$

Dopo quanto tempo la velocità della sbarretta sarà ridotta alla metà della velocità iniziale?

Sol



$$\phi_S(B) = \int_S \vec{B}\,\hat{n}\,ds = \int_S B\,ds =$$

$$= B \cdot \text{Superficie} = B \cdot L \cdot (d - x)$$

$$\Rightarrow \frac{d\phi_S(B)}{dT} = -BLV(t)$$

$$\Rightarrow \xi = -\frac{d\phi_S(B)}{dT} = BLV(t)$$

$$\Rightarrow \quad \xi = -\frac{\partial \phi_S(B)}{\partial T} = B\,L\,V(t)$$

$$\Rightarrow \quad i_{ind} = \frac{\xi}{R} = \frac{B\,L\,V(t)}{R}$$

$$dF = i\, d\ell \wedge B$$

$$\Rightarrow \quad F = i\,L \cdot B = \frac{B^2 L^2\, V(t)}{R}$$

So che

$$F = m \cdot a$$

Dunque, uguagliando si ha che:

$$\frac{B^2 L^2\, V(t)}{R} = m \cdot a$$

Dato che l'accelerazione è la derivata della velocità ho che

$$a = \frac{dV}{dt}$$

$$\Rightarrow \quad \frac{B^2 L^2\, V(t)}{R} = m \frac{dV}{dt}$$

$\Rightarrow$

$$\frac{B^2L^2}{mR}\,dt = \frac{1}{V(t)}\,dV$$

$\Rightarrow$

$$\int_0^t \frac{B^2L^2}{mR}\,dt = \int_{V_0}^{\frac{1}{2}V_0} \frac{1}{V_{(t)}}\,dV$$

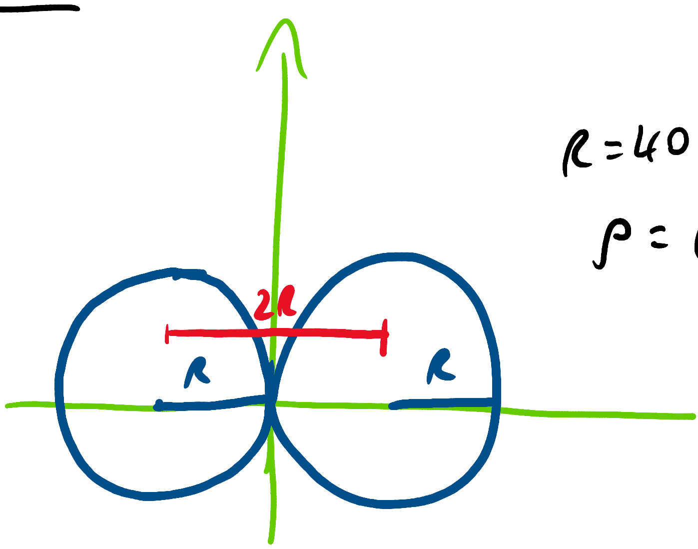
$\Rightarrow$

$$\frac{B^2L^2}{mR}\,t = \ln\left(\tfrac{1}{2}V_0 - V_0\right) = \ln\left(\tfrac{1}{2}V_0 \cdot \tfrac{1}{V_0}\right) = \ln\left(\tfrac{1}{2}\right)$$

$$\Rightarrow \quad t = \ln\left(\tfrac{1}{2}\right) \cdot \frac{mR}{B^2L^2}$$

$\blacksquare$

Es 2



$R = 40\ cm$

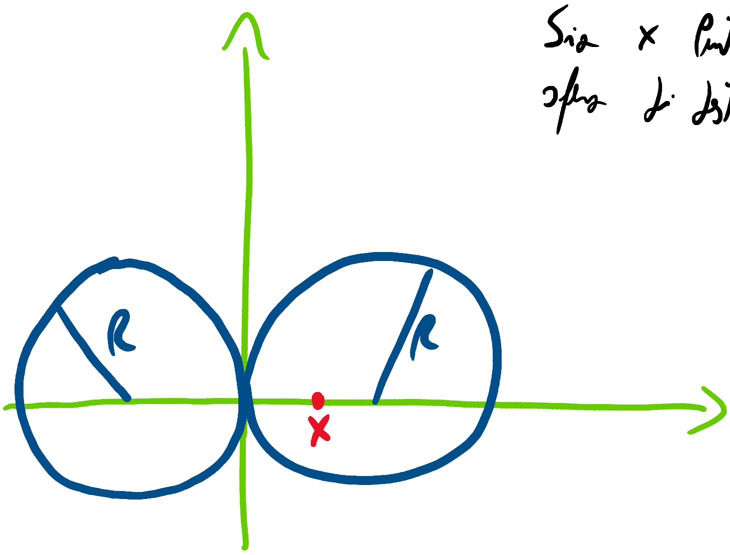
$\rho = 6 \cdot 10^{-7}\ C/m^3$

a) Utilizzare il teorema di Gauss per scrivere l'espressione del modulo del campo elettrico lungo l'asse delle x all'interno della sfera di destra e poi calcolare il valore in $x = R$

b) Utilizzare il teorema di Gauss per scrivere l'espressione del campo elettrico lungo l'asse y e poi calcolare il valore in $y = R$

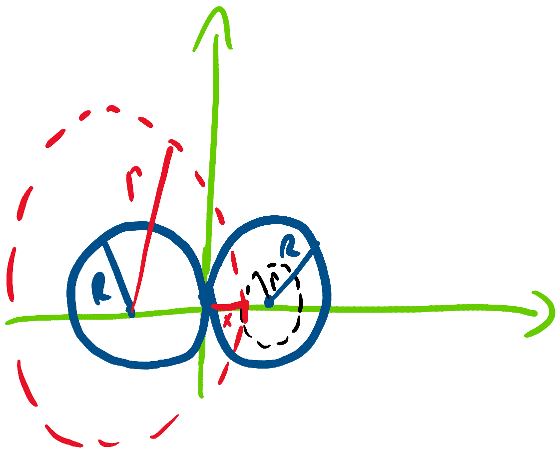
<u>SOL</u>

a)



Sia x punto generico sulla sfera di destra

Costruiamo ora due superfici di GAUSS: una per la sfera di sinistra
una per la sfera di destra



Cominciamo con quella di DESTRA

$$\phi_S(E_{dx}) = \int_S E_{dx}\cdot S = \frac{Q_{int}}{\varepsilon_0}$$

$\Rightarrow$

$$E_{dx}\cdot 4\pi r^2 = \frac{Q_{int}}{\varepsilon_0}$$

$\Rightarrow$ $r = R - x$

$$E_{dx}\cdot 4\pi (R-x)^2 = \frac{Q_{int}}{\varepsilon_0}$$

$$Q_{int} = \rho \cdot V$$

volume che contiene Q_{int}

$\Rightarrow$ raggio sarà tra 0 e r

$\Rightarrow$ raggio sarà tra 0 e $(R-x)$

$$V = \frac{4}{3}\pi r^3$$

$$\Rightarrow dV = 4\pi r^2 dr \Rightarrow V = \int_0^{R-x} 4\pi r^2 dr = 4\pi \left(\frac{r^3}{3}\right)\Bigg|_0^{R-x} = \frac{4}{3}\pi (R-x)^3$$

contiene Q_{int}

$\Rightarrow$

$Q_{int} = \rho \cdot V$

$\Rightarrow$

$$E_{dx} \cdot 4\pi (R-x)^2 = \frac{Q_{int}}{\varepsilon_0} = \frac{\rho \cdot V}{\varepsilon_0} =$$

$$= \frac{\rho \cdot \frac{4}{3}\pi (R-x)^3}{\varepsilon_0}$$

$\Rightarrow$

$$E_{dx} = \frac{\rho \cdot \cancel{\frac{4}{3}\pi} (R-x)^3}{\cancel{4\pi} (R-x)^2 \varepsilon_0} = \frac{\rho \cdot (R-x)}{3 \varepsilon_0}$$

Vediamo ora E_{sx}

$$\phi_S(E_{sx}) = \int_S E_{sx} \, dS = \frac{Q_{int}}{\varepsilon_0}$$

$\Rightarrow$

$$E_{sx} \cdot 4\pi r^2 = \frac{Q_{int}}{\varepsilon_0}$$

$Q_{int} = Q_{TOT}$ perché la nostra superficie di Gauss riesce a racchiudere tutta la sfera di S_x

$\Rightarrow$ $r = R + x$

$$E_{sx} \cdot 4\pi (R+x)^2 = \frac{Q_{int}}{\varepsilon_0} = \frac{Q_{TOT}}{\varepsilon_0}$$

Volume che contiene Q_{TOT} $\Rightarrow$ raggio compreso tra 0 ed R

$\Rightarrow$

$$E_{sx} = \frac{Q_{TOT}}{4\pi \varepsilon_0 (R+x)^2} = \frac{\rho \cdot V}{4\pi \varepsilon_0 (R+x)^2}$$

$\Rightarrow$

$$E_{sx} = \frac{\rho \cdot \frac{4}{3}\pi R^3}{4\pi \varepsilon_0 (R+x)^2} = \frac{\rho R^3}{3 \varepsilon_0 (R+x)^2}$$

$\Rightarrow$

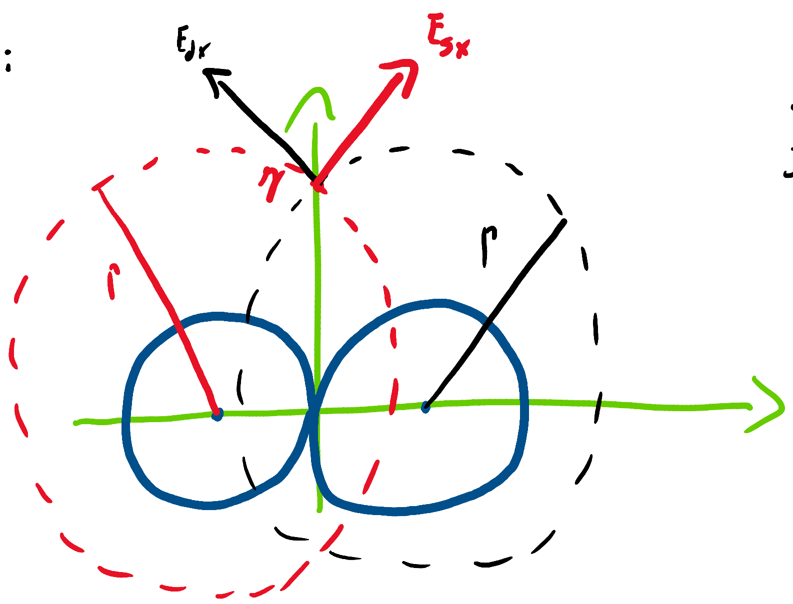
$$E_{tot} = E_{sx} - E_{dx} = \frac{\rho R^3}{\ldots} - \frac{\rho (R-x)}{3 \varepsilon_0}$$

$$\Rightarrow E_{TOT} = E_{sx} - E_{dx} = \frac{\rho R^3}{3\varepsilon_0 (R+x)^2} - \frac{\rho (R-x)}{3\varepsilon_0}$$

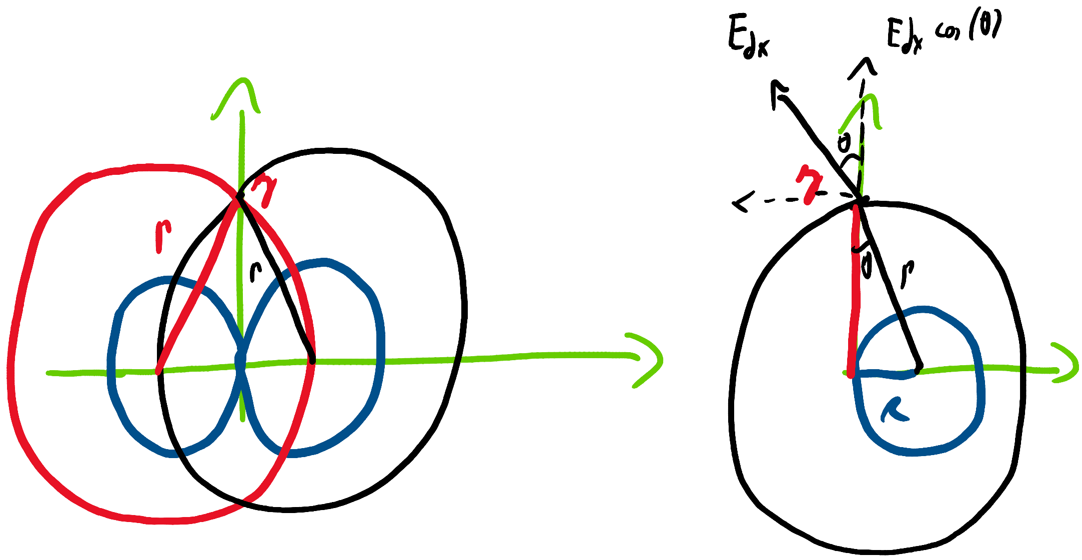
Se $x=R$ si ha che

$$E_{TOT}(x=R) = \frac{\rho R^3}{3\varepsilon_0 (R+R)^2} = \frac{\rho R}{12\varepsilon_0}$$

b):



Sia y punto generico sull'asse delle y



Per simmetria sopravvive solo la componente sull'asse delle y

$$\Rightarrow E_{TOT} = E_{dx}\cos(\theta) + E_{sx}\cos(\theta) \overset{\text{Per simmetria}}{=} 2E_{dx}\cos(\theta)$$

$\Rightarrow$

$$E_{TOT} = E_{dx}\cos(\theta) + E_{sx}\cos(\theta) \overset{\downarrow}{=} 2E_{dx}\cos(\theta)$$

Troviamo pertanto E_{dx}

$$\phi_S(E_{dx}) = \int_S E_{dx}\, dS = \frac{Q_{int}}{\varepsilon_0}$$

$\Rightarrow$

$$E_{dx} \cdot \text{Superficie} = \frac{Q_{int}}{\varepsilon_0}$$

$\Rightarrow$

$$E_{dx} \cdot 4\pi r^2 = \frac{Q_{int}}{\varepsilon_0} = \frac{Q_{TOT}}{\varepsilon_0}$$

$\Rightarrow$

$$E_{dx} = \frac{Q_{TOT}}{4\pi\varepsilon_0 r^2} = \frac{\rho \cdot V}{4\pi\varepsilon_0 r^2} = \frac{\rho \cdot \frac{4}{3}\pi R^3}{4\pi\varepsilon_0 r^2} =$$

$$= \frac{\rho \cdot R^3}{3\varepsilon_0 r^2} \overset{r=\sqrt{R^2+\eta^2}}{=} \frac{\rho \cdot R^3}{3\varepsilon_0 (R^2+\eta^2)}$$

$\Rightarrow$

$$E = E_{dx}\cos(\theta) = \frac{\rho \cdot R^3}{3\varepsilon_0 (R^2+\eta^2)}\cos(\theta)$$

Poiché $\eta = r\cos(\theta) \Rightarrow \cos(\theta) = \frac{\eta}{r} = \frac{\eta}{\sqrt{R^2+\eta^2}}$

$\Rightarrow$

$$E = \frac{\rho R^3}{} \cdot \frac{\eta}{} = \frac{\rho R^3 \eta}{}$$

$$E = \frac{\rho R^3}{3\varepsilon_0 (R^2+z^2)} \cdot \frac{z}{\sqrt{R^2+z^2}} = \frac{\rho R^3 z}{3\varepsilon_0 (R^2+z^2)^{3/2}}$$

$\Rightarrow$

$$E_{TOT} = 2E = \frac{2}{3} \cdot \frac{\rho R^3}{\varepsilon_0} \cdot \frac{z}{(R^2+z^2)^{3/2}}$$

se $z = R$, si ha

$$E_{TOT}(z=R) = \frac{2}{3} \frac{\rho R^4}{\varepsilon_0} \cdot \frac{1}{(2R^2)^{3/2}} = \frac{2}{3} \cdot \frac{1}{\varepsilon_0} \cdot \rho \cdot \frac{R^4}{2\sqrt{2}\, R^3} =$$

$$= \frac{1}{3\sqrt{2}} \cdot \frac{1}{\varepsilon_0} \cdot \rho \cdot R$$

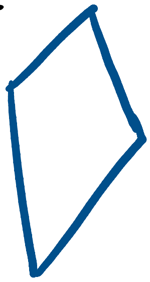
Es 3

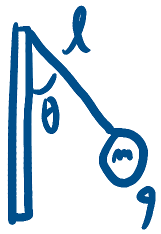
* Da fare *

Appello 7/12/2022

martedì 16 gennaio 2024 15:55

Es 1





$\theta = 30°$

$m = 1{,}12\ mg$

$q = 19{,}7\ nC$

$l = 2\ cm$

EQUILIBRIO

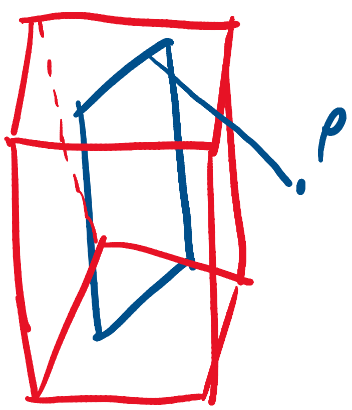
a) Determinare col teorema di Gauss l'espressione del modulo del campo elettrico generato dal foglio e si indichi direzione e verso nel punto in cui si trova la piccola sfera

b) Determinare σ del foglio

c) Assumendo che la carica sul foglio svanisca all'istante t_0, si determini la velocità della sfera quando passa per il punto più basso

SOL

a)



$$\int_S E\,ds = \frac{Q_{int}}{\varepsilon_0}$$

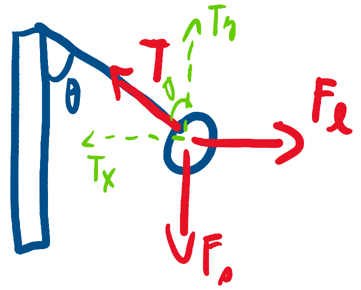
$$\Rightarrow \quad E \cdot S = \frac{Q_{int}}{\varepsilon_0}$$

moltiplico per due perché ho due facce laterali del mio cubo di lato L

$$\Rightarrow \quad E \cdot 2L^2 = \frac{Q}{\varepsilon_0} = \frac{\sigma \cdot S_{superficie}}{\varepsilon_0} = \frac{\sigma \cdot L^2}{\varepsilon_0}$$

$$\Rightarrow \quad E = \frac{\sigma}{2\varepsilon_0}$$

b):



$$F_{tot} = 0$$

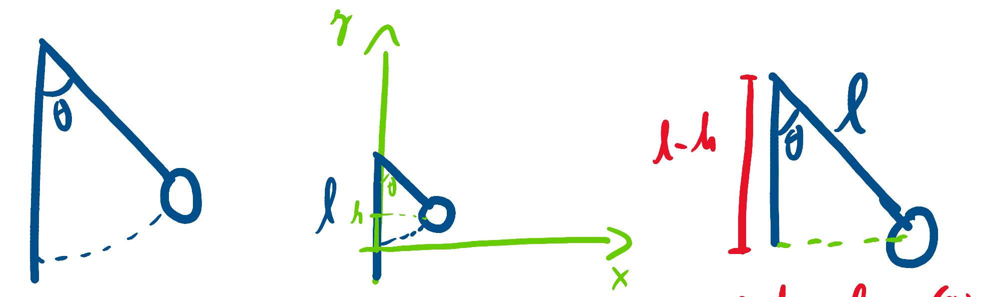
$$\Rightarrow \begin{cases} F_{x\,tot} = 0 \\ F_{y\,tot} = 0 \end{cases} \Rightarrow \begin{cases} F_e - T_x = 0 \\ T_y - F_p = 0 \end{cases} \Rightarrow \begin{cases} F_e = T_x \\ T_y = F_p \end{cases}$$

$$\Rightarrow \begin{cases} E \cdot q - T\sin(\theta) = 0 \\ T\cos(\theta) = mg \end{cases} \Rightarrow \begin{cases} \frac{\sigma}{2\varepsilon_0} \cdot q = T\sin(\theta) \\ mg = T\cos(\theta) \end{cases}$$

$$\Rightarrow \quad \frac{\sigma \cdot q}{2\varepsilon_0 mg} = \tan(\theta)$$

$$\Rightarrow \quad \sigma = tg(\theta) \cdot \frac{2\varepsilon_0 m g}{q}$$

c)



$$l - h = l\cos(\theta)$$

$$\Rightarrow h = l - (l - h)$$
$$= l - l\cos(\theta)$$

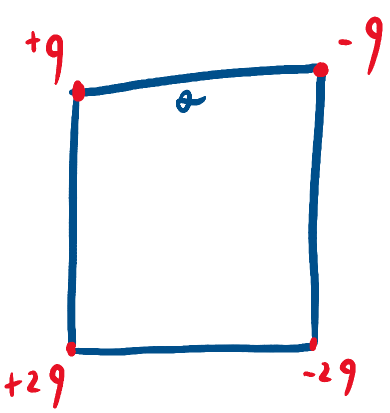
$$\Delta E_c + \Delta U = L_{f.c.}$$

$$\Rightarrow \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 + m g h_f - m g h_i = 0$$

$$\Rightarrow \frac{1}{2} \cancel{m} v_f^2 = \cancel{m} g h_i$$

$$\Rightarrow v_f = \sqrt{2 g h_i} = \sqrt{2g(l - l\cos(\theta))}$$

Es 3

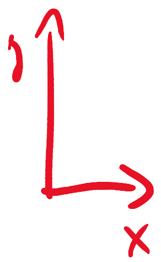


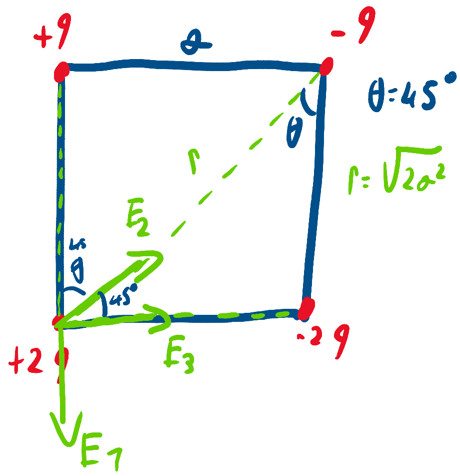
$q = 1{,}13\ \mu C$

$a = 15{,}2\ cm$

Calcolare la forza elettrica risultante agente sulla carica in basso a sinistra

Sol





E_2 diretto verso $-q$ perché è carica negativa

E_3 stesso andamento di E_2

$$E_{TOT} = E_1 + E_2 + E_3$$

$$E_2 = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{-q}{2a^2} \Rightarrow |E_2| = \frac{q}{4\pi\varepsilon_0 2a^2}$$

$$E_3 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{-2q}{a^2} \Rightarrow |E_3| = \frac{2q}{4\pi\varepsilon_0 a^2}$$

$$E_1 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{q}{a^2} \Rightarrow |E_1| = \frac{q}{4\pi\varepsilon_0 a^2}$$

$$E_1 = \left(0, -\frac{1}{4\pi\varepsilon_0}\frac{q}{a^2}\right)$$

$$E_2 = \left(E_2\cos(\theta), E_2\sin(\theta)\right) = \left(\frac{q}{4\pi\varepsilon_0 2a^2}\cdot\frac{\sqrt{2}}{2}, \frac{q}{4\pi\varepsilon_0 2a^2}\cdot\frac{\sqrt{2}}{2}\right)$$

$$E_3 = \left(\frac{2q}{4\pi\varepsilon_0 a^2}, 0\right)$$

$$E_{TOT} = \left(\frac{q}{16\pi\varepsilon_0 a^2}\cdot\sqrt{2} + \frac{2q}{4\pi\varepsilon_0 a^2}, \frac{q\sqrt{2}}{16\pi\varepsilon_0 a^2} - \frac{1}{4\pi\varepsilon_0}\frac{q}{a^2}\right)$$

$$F_{TOT} = E_{TOT} \cdot 2q$$

$\Rightarrow$

$$F_{TOT} = \left(\frac{q^2\sqrt{2}}{8\pi\varepsilon_0 a^2} + \frac{q^2}{\pi\varepsilon_0 a^2} \, , \, \frac{q^2\sqrt{2}}{8\pi\varepsilon_0 a^2} - \frac{q^2}{2\pi\varepsilon_0 a^2} \right) =$$

$$= \left(\frac{q^2\sqrt{2} + 8q^2}{8\pi\varepsilon_0 a^2} \, , \, \frac{q^2\sqrt{2} - 4q^2}{8\pi\varepsilon_0 a^2} \right) =$$

$$= \left(\frac{q^2(\sqrt{2}+8)}{8\pi\varepsilon_0 a^2} \, , \, \frac{q^2(\sqrt{2}-4)}{8\pi\varepsilon_0 a^2} \right) =$$

$$= \frac{q^2}{8\pi\varepsilon_0 a^2} \left(\sqrt{2}+8 \, , \, \sqrt{2}-4 \right)$$

$$|F_{TOT}| = \sqrt{F_{TOT_x}^2 + F_{TOT_y}^2} = \frac{q^2}{8\pi\varepsilon_0 a^2} \sqrt{2 + 64 + 16\sqrt{2} + 2 + 16 - 8\sqrt{2}} =$$

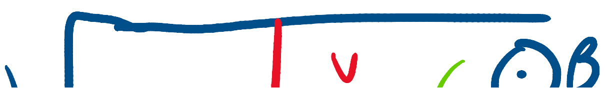
$$= \frac{q^2}{8\pi\varepsilon_0 a^2} \sqrt{84 + 8\sqrt{2}}$$

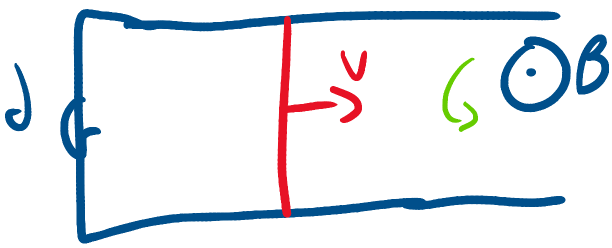
$\blacksquare$

Es 2

$\xi = 0{,}1\,V$

$d = 50\,cm$





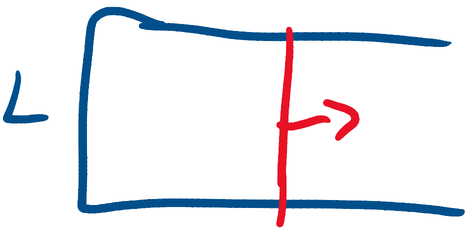
$d = 50\ \text{cm}$

$B = 1\ \text{T}$

a) Determinare il verso di circolazione della i_{ind}

b) Dimostrare che la velocità della barretta tende ad un valore costante

Sol



$x(t) = S_0 + v \cdot t$

$$\Phi_S(B) = \int_S B\, dS = B \cdot S_{superficie} = B \cdot L\,(S_0 + vT)$$

$\Rightarrow$

$$\frac{\partial \Phi_S(B)}{\partial t} = BLV$$

Si muove verso sinistra

$$\varepsilon = -BLV \Rightarrow 0,1 = -BLV \Rightarrow V = -\frac{0,1}{BL}$$

$$i_{ind} = \frac{\varepsilon}{R} = -\frac{BLV}{R} < 0 \quad \Rightarrow \text{gira in senso orario}$$

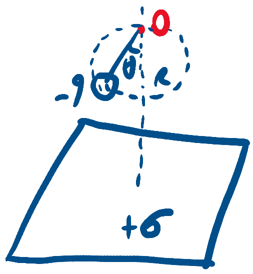
b) $v(t) = \lim \ -\frac{\varepsilon}{\ldots} = -\frac{0,1}{\ldots} = 0,2$

h) $V(t) = \lim_{t \to +\infty} -\frac{\xi}{BL} = -\frac{0,1}{BL} = 0,2$

Appello 18/07/2022

mercoledì 17 gennaio 2024 11:09

Es 1



$q = -2{,}95 \cdot 10^{-7} C$

$m = 10^{-4} g$

$R = 2 \cdot 10^{-7} m$

$V = 0{,}1 \ m/s$

$\sigma = 3 \cdot 10^{-7} \ C/m^2$

i) Calcolare il periodo di rotazione

ii) il modulo della tensione del filo

iii) il valore di θ

Dimostrare che la forza di gravità agente su m è trascurabile

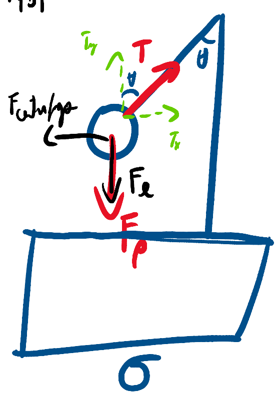
SoL

i) $w = \frac{2\pi}{T}$ $\qquad w = \frac{V}{r}$

$\Rightarrow \quad T = \frac{2\pi}{w} = \frac{2\pi}{V} \cdot R$

Periodo

ii) $F_{TOT} = 0$



$T_x = T \cdot \sin(\theta)$

$T_y = T \cdot \cos(\theta)$

$F_e = E \cdot q = \frac{\sigma}{2\varepsilon_0} q$

$\{ \ T_x - F_{centrifuga} = 0$

$\{ \ T \cdot \sin(\theta) = m w^2 R = m \frac{V^2}{R^2} \cdot R = m \frac{V^2}{R}$

$$\begin{cases} F_{TOT_x} = 0 \\ F_{TOT_y} = 0 \end{cases} \Rightarrow \begin{cases} T_x - F_{centrifuga} = 0 \\ T_y - F_p - F_e = 0 \end{cases} \Rightarrow \begin{cases} T \cdot \sin(\theta) = m\,\omega^2 R = m\,\frac{v^2}{R^2} \cdot R = m\,\frac{v^2}{R} \\ T \cdot \cos(\theta) = mg + \frac{\sigma}{2\varepsilon_0}\,q \end{cases}$$

$$\Rightarrow tg(\theta) = m\,\frac{v^2}{R} \cdot \left(\frac{1}{mg + \frac{\sigma}{2\varepsilon_0}\,q} \right)$$

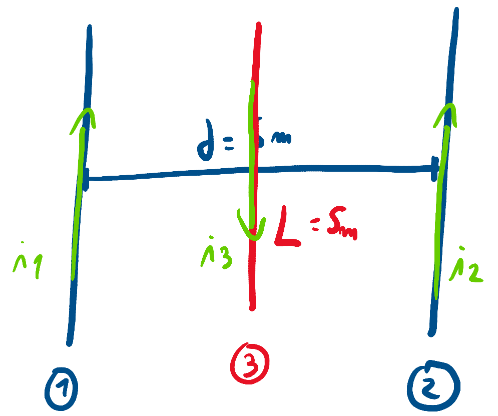
$$\Rightarrow \theta = tg^{-1}\left(\frac{m\,\frac{v^2}{R}}{mg + \frac{\sigma q}{2\varepsilon_0}} \right)$$

Dalla 1° eq del sistema

$$\Rightarrow T = m\,\frac{v^2}{R} \cdot \left(\frac{1}{\sin(\theta)} \right)$$

F_p è trascurabile perché è MOLTO PICCOLA

Es 2



Determinare

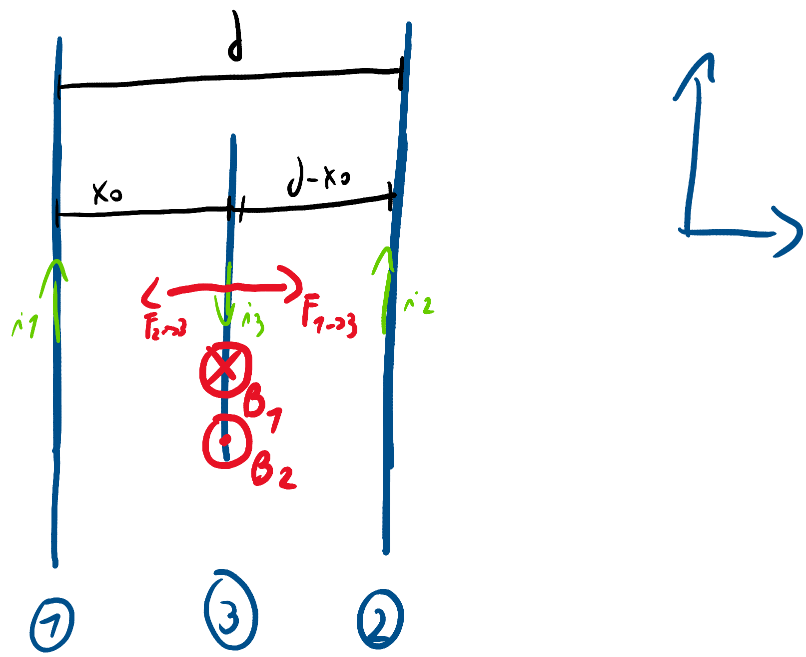
a): la distanza x_0 dal filo 1 alla quale il filo 3 risulterà in equilibrio risulta in equilibrio

b): il lavoro compiuto dalla Forza Magnetica se il filo 3 si sposta di $\Delta = 20$ cm verso il filo 2

Che tipo di equilibrio è quello in a)?

Cosa cambia se i_3 scorre come i_2 e i_1?

<u>SOL</u>



$B_{tot} = B_1 - B_2$

sul filo 3

B generato dal filo 1

B generato dal filo 2

$$dF = i\, dl \wedge B$$

$\Rightarrow$

$$F = i\, L\, B$$

$F_{TOT} = 0$

$\Rightarrow$

$$F_{1\to 3} - F_{2\to 3} = 0 \quad \Rightarrow \quad F_{1\to 3} = F_{2\to 3}$$

$$F_{1\to 3} = i_3 L \frac{\mu_0 i_1}{2\pi x_0}$$

$$F_{2\to 3} = i_3 L \frac{\mu_0 i_2}{2\pi (d-x_0)}$$

Dunque

$$\cancel{i_3} \cancel{L} \frac{\cancel{\mu_0} i_1}{\cancel{2}\cancel{\pi} x_0} = \cancel{i_3} \cancel{L} \frac{\cancel{\mu_0} i_2}{\cancel{2}\cancel{\pi}(d-x_0)}$$

$$\Rightarrow \frac{i_1}{x_0} = \frac{i_2}{d-x_0}$$

$$\Rightarrow \frac{x_0}{d-x_0} = \frac{i_1}{i_2} \quad \Rightarrow \quad x_0 = \frac{i_1}{i_2}(d-x_0)$$

$$\Rightarrow x_0 = \frac{i_1}{i_2} d - \frac{i_1}{i_2} x_0 \quad \Rightarrow \quad x_0 + \frac{i_1}{i_2} x_0 = \frac{i_1}{i_2} d$$

$$\Rightarrow x_0 \left(1 + \frac{i_1}{i_2}\right) = \frac{i_1}{i_2} d$$

$\Rightarrow$

$$x_0 = \frac{\frac{i_1}{i_2} \cdot d}{1 + \frac{i_1}{i_2}}$$

i_2

h)

$$L = \int_{\gamma} F\,dl = \int_{x_0}^{x_0+\Delta} F\,dl = \int_{x_0}^{x_0+\Delta} F_{1\to 3} - F_{2\to 3}\,dl =$$

Tragitto da x_0 a $x_0+\Delta$

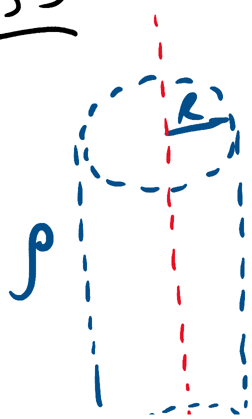
$$= \int_{x_0}^{x_0+\Delta} L\,\frac{\mu_0 i_1 i_3}{2\pi x} - L\,\frac{\mu_0 i_2 i_3}{2\pi (d-x)}\,dx =$$

$$= L\,\frac{\mu_0 i_3}{2\pi} \int_{x_0}^{x_0+\Delta} \frac{i_1}{x} - \frac{i_2}{d-x}\,dx =$$

$$= \frac{L\mu_0 i_3}{2\pi}\left(i_1 \ln(x)\Big|_{x_0}^{x_0+\Delta} + i_2 \ln(d-x)\Big|_{x_0}^{x_0+\Delta} \right) =$$

$$= \frac{L\mu_0 i_3}{2\pi}\left(i_1 \ln\left(\frac{x_0+\Delta}{x_0}\right) + i_2 \ln\left(\frac{d-x_0-\Delta}{d-x_0}\right)\right)$$

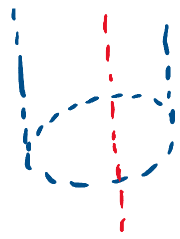
Es 3



$R = 10\,cm$

$$\rho = \frac{\rho_0}{R}\cdot r$$

r è la distanza dall'asse del cilindro



Calcolare ΔV tra $r=0$ e $r=R$

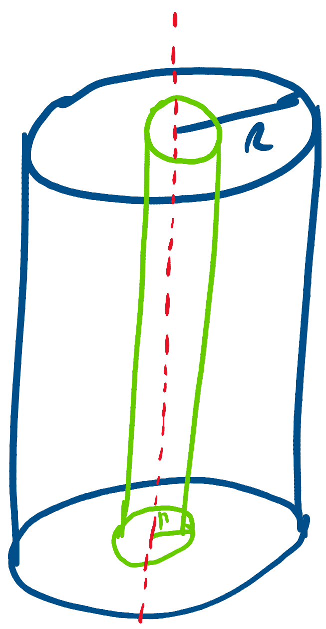
<u>SOL</u>

$$-\Delta V = \int_a^b E\, dl$$

$$V_{(a)} - V_{(b)} = \int_a^b E\, dl$$

$$V_{(0)} - V_{(R)} = \int_0^R E\, dl$$

Dobbiamo trovare E, proviamo con GAUSS



$$\int_S E\, dS = \frac{Q_{int}}{\varepsilon_0}$$

$$\Rightarrow \quad E \cdot 2\pi r h = \frac{Q_{int}}{\varepsilon_0}$$

Volume di un cilindro : $V = \pi r^2 h$

$\Rightarrow dV = 2\pi r h \, dr$

volume che contiene la sua Q_{int}

ovvero il cilindro che varia il raggio tra 0 e r

Non conosco Q_{int}

Però vale che

$$Q_{int} = \rho V$$

$$\Rightarrow \quad Q_{int} = \int_0^r \rho \cdot dV = \int_0^r \frac{\rho_0}{R} \cdot r \cdot 2\pi r h \, dr$$

integro anche ρ perché in questo esercizio ρ dipende da r

$$= \frac{2\pi \rho_0 h}{R} \int_0^r r^2 \, dr =$$

$$= \frac{2\pi h \rho_0}{R} \frac{r^3}{3}$$

$$\Rightarrow \quad Q_{int} = \frac{2\pi h \rho_0 r^3}{3R}$$

Dunque

$$E \cdot \cancel{2}\cancel{\pi} r \cancel{h} = \frac{Q_{int}}{\varepsilon_0} = \frac{\cancel{2}\cancel{\pi}\cancel{h}\rho_0 r^3}{3R\varepsilon_0}$$

$$\Rightarrow \quad E = \frac{\rho_0 r^2}{3R\varepsilon_0}$$

Dunque

$$V_{(0)} - V_{(R)} = \int_0^R E \, dr = \int_0^R \frac{\rho_0 r^2}{3R\varepsilon_0} \, dr =$$

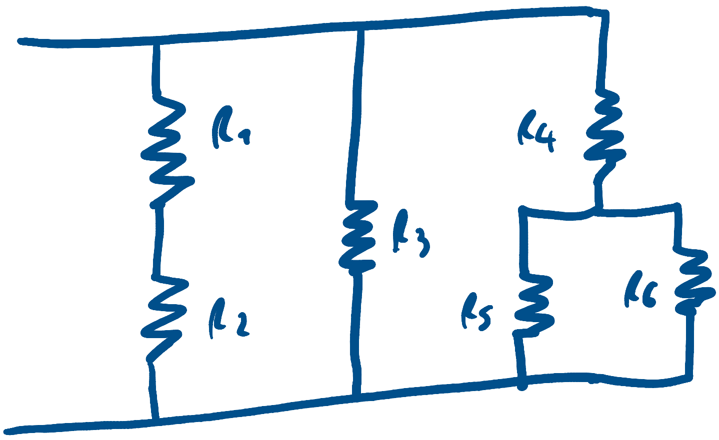
$$= \frac{\rho_0}{3R\varepsilon_0} \frac{R^3}{3} = \frac{\rho_0 R^2}{9\varepsilon_0}$$

$\blacksquare$

Appello 14/03/2023

mercoledì 17 gennaio 2024 15:07

Es 3



$R_1 = 10$

$R_2 = 20$

$R_3 = 15$

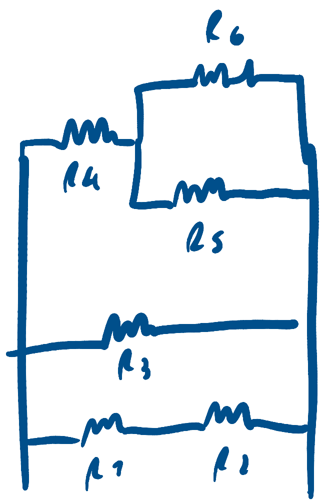
$R_4 = 5$

$R_5 = 15$

$R_6 = 30$

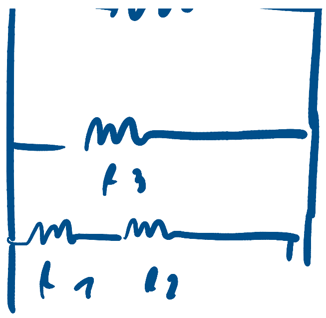
Determinare R_{eq}

SOL



$$\frac{1}{R_{eq_{56}}} = \frac{1}{R_6} + \frac{1}{R_5} \Rightarrow R_{eq_{56}} = \left(\frac{1}{R_6} + \frac{1}{R_5}\right)^{-1}$$

$$R_{eq_{456}} = R_4 + R_{eq_{56}}$$



$$R_{eq_{12}} = R_1 + R_2$$

$$\frac{1}{R_{eq_{123456}}} = \frac{1}{R_{eq_{456}}} + \frac{1}{R_3} + \frac{1}{R_{eq_{12}}}$$

$$\Rightarrow R_{eq_{TOT}} = \left(\frac{1}{R_{eq_{456}}} + \frac{1}{R_3} + \frac{1}{R_{eq_{12}}} \right)^{-1}$$

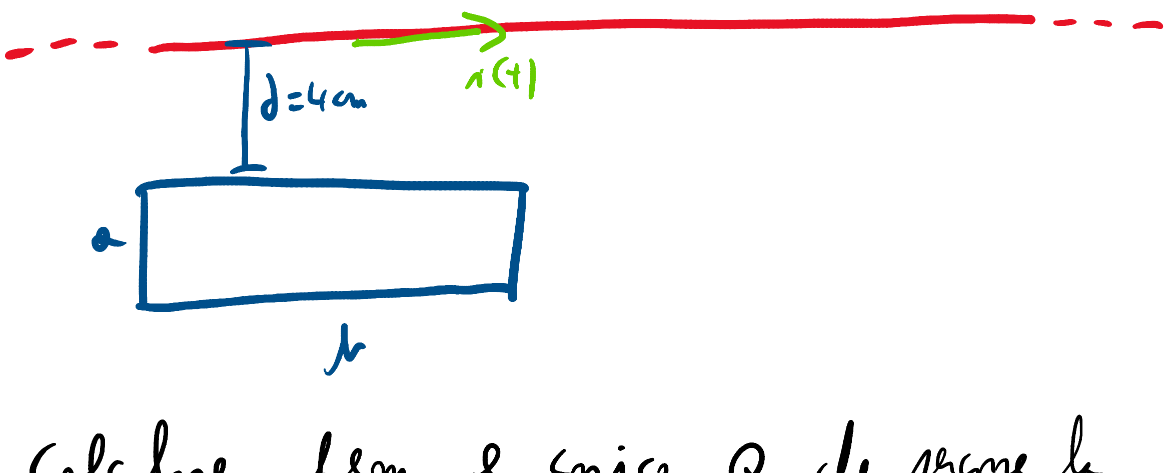
E, 2

$i(t) = i_0 \, e^{-\frac{t}{\tau}}$

$a : 6\,cm$

$b : 12\,cm$

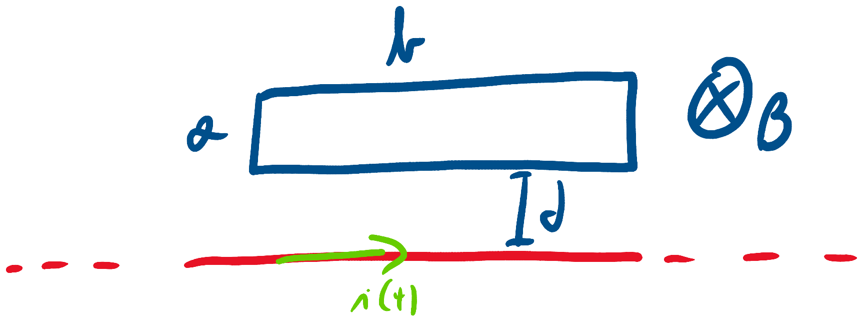
$R = 2\,\Omega$

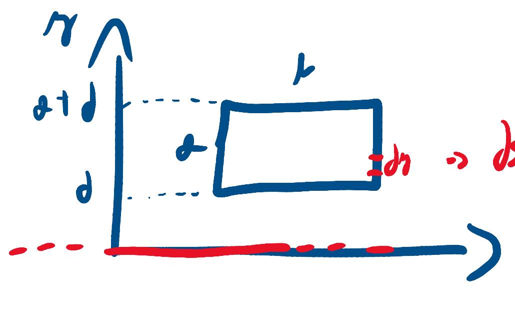


Calcolare fem e carica Q che passa la

Calcolare f.e.m. e carica Q che passa la spira nell'intervallo di tempo da 0 a +∞

Sol





$\Rightarrow ds = b\,dy$

$$\phi_s(B) = \int_S B\,ds = \int_S \frac{\mu_0 i}{2\pi y}\,ds = \int_a^{a+d} \frac{\mu_0 i}{2\pi y}\,b\,dy$$

$$= \frac{\mu_0 i b}{2\pi} \ln\left(\frac{a+d}{a}\right) = \frac{\mu_0 b}{2\pi} i_0 \cdot e^{-\frac{t}{\tau}} \ln\left(\frac{a+d}{a}\right)$$

$$\frac{d\phi_s(B)}{dt} = \frac{\mu_0 b}{2\pi} i_0 \ln\left(\frac{a+d}{a}\right) e^{-\frac{t}{\tau}} \cdot \left(-\frac{1}{\tau}\right)$$

$$\Rightarrow \xi = -\frac{d\phi_s(B)}{dt} = \frac{\mu_0 i_0 b}{2\pi\tau} \ln\left(\frac{a+d}{a}\right) e^{-t/\tau}$$

$$\Rightarrow i_{ind} = \frac{\xi}{\cdot} = \frac{\mu_0 i_0 b}{2\pi\tau\,\cdot} \cdot \ln\left(\frac{a+d}{a}\right) e^{-\frac{t}{\tau}}$$

$$\Rightarrow i_{ind} = \frac{\xi}{R} = \frac{\mu_0 i_0 h}{2\pi\tau R} \cdot \ln\left(\frac{\quad}{a}\right) e$$

Siccome vale che

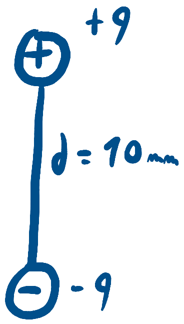
$$i = \frac{dQ}{dt}$$

$$\Rightarrow dQ = i\,dt$$

$$\Rightarrow Q = \int_t i\,dt = \int_0^{+\infty} \frac{\mu_0 i_0 h}{2\pi\tau R} \ln\left(\frac{a+d}{a}\right) e^{-\frac{t}{\tau}}\,dt$$

Es 1

$|q| = 1\,\mu C$



Utilizzando le coordinate polari scrivere le espressioni per E_r, E_θ, E_ϕ

Calcolare E quando $r = 10\,m$ e $\theta = \frac{\pi}{2}$ e ϕ qualsiasi

Sol

Sapendo che ΔV di un dipolo è

$$\Delta V = \frac{p\cos(\theta)}{4\pi\varepsilon_0 r^2}$$

$$\Delta V = \frac{p \cos(\theta)}{4\pi \varepsilon_0 r^2}$$

e il gradiente è dato da

$$\nabla = \left(\frac{\partial}{\partial r} , \frac{1}{r} \frac{\partial}{\partial \theta} , \frac{1}{r \sin(\theta)} \frac{\partial}{\partial \varphi} \right)$$

Si ha che

$$E = -\nabla V$$

$$\Rightarrow \quad E = \begin{cases} E_r = -\left(\frac{p \cos(\theta)}{4\pi \varepsilon_0} \cdot \left(-\frac{2}{r^3} \right) \right) = \frac{p \cos(\theta)}{2\pi \varepsilon_0} \cdot \frac{1}{r^3} \\ E_\theta = -\frac{1}{r} \left(-\frac{p \sin(\theta)}{4\pi \varepsilon_0 r^2} \right) = \frac{p \sin(\theta)}{4\pi \varepsilon_0 r^3} \\ E_\varphi = 0 \end{cases}$$

$$\Rightarrow \quad E = \sqrt{E_r^2 + E_\theta^2 + E_\varphi^2} = \dots = \frac{p}{4\pi \varepsilon_0 r^3} \sqrt{1 + 3\cos^2(\theta)}$$

Se $r = 10$ e $\theta = \frac{\pi}{2}$, si ha:

$$E = \begin{cases} E_r = 0 \\ E_\theta = \frac{p}{4\pi \varepsilon_0 (10)^3} \\ E_\varphi = 0 \end{cases}$$

$$\Rightarrow \quad E = \sqrt{E_r^2 + E_\theta^2 + E_\varphi^2} = E_\theta = \frac{p}{4\pi \varepsilon_0 (10)^3} =$$

$p = q \cdot d$

momento

$$= \frac{q \cdot d}{4\pi \varepsilon_0 (10)^3}$$

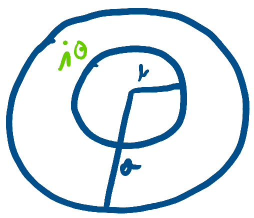
$$1 = \frac{1}{4\pi\varepsilon_0 (10)^3}$$

Momento
Dipolo
Elettrico

Appello 27/01/2023

giovedì 18 gennaio 2024 10:21

Es 1



Determinare il campo magnetico alla distanza radiale r dall'asse con

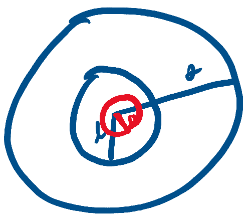
(i) $r < b$

(ii) $b \le r \le a$

(iii) $r > a$

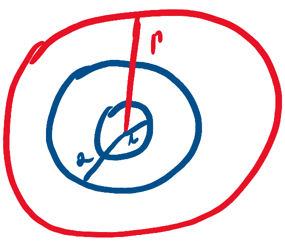
SOL

(i) $(r < b)$ Uso AMPERE



$$\oint_\gamma B\, dl = \mu_0 \underbrace{i_{conc}}_{=0} \Rightarrow B = 0$$

(iii) $(r > a)$

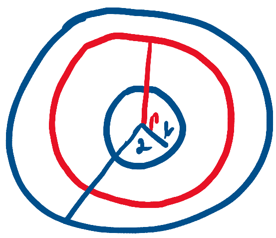


$$\oint_\gamma B\,dl = \mu_0 \, i_{conc}$$

$$\Rightarrow B \, 2\pi r = \mu_0 \, i$$

$$\Rightarrow B = \frac{\mu_0 \, i}{2\pi r}$$

(ii) $(b \leq r \leq a)$



$$\oint_\gamma B\,dl = \mu_0 \, i_{conc}$$

$$B \cdot 2\pi r = \mu_0 \, i_{conc}$$

$$\Rightarrow B = \frac{\mu_0 \, i_{conc}}{2\pi r}$$

Dobbiamo riscrivere meglio i_{conc}

Sapendo che

$$J = \frac{di}{dS} \Rightarrow di = J \, dS$$

$$\Rightarrow i_{conc} = J \cdot S$$

Superficie che contiene i_{conc}

Quella di un cerchio con raggio che va da $r = b$ a $r = r$

$$\Rightarrow i_{conc} = J \cdot \pi (r^2 - b^2)$$

Formula generale dell'area di un cerchio

$\downarrow$

$$S = \pi r^2$$

$$dS = 2\pi r \, dr$$

$$S = \int_b^r 2\pi r \, dr = 2\pi \frac{r^2}{2}\Big|_b^r = \pi (r^2 - b^2)$$

Dunque

$$B = \frac{\mu_0 \cdot J \cdot \pi (r^2 - b^2)}{2 \pi r}$$

Scrivendo meglio J :

$$J = \frac{di}{dS} \Rightarrow J = \frac{i_{TOT}}{S_{TOT}} = \frac{i}{\pi (a^2 - b^2)}$$

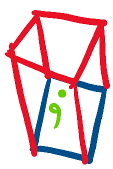
Dunque

$$B = \frac{\mu_0}{2 \pi r} \cdot \frac{i}{\cancel{\pi} (a^2 - b^2)} \cdot \cancel{\pi} (r^2 - b^2) = \frac{\mu_0 \, i \, (r^2 - b^2)}{2 \pi r \, (a^2 - b^2)}$$

Es 3

Calcolare il flusso del campo elettrico

Sol



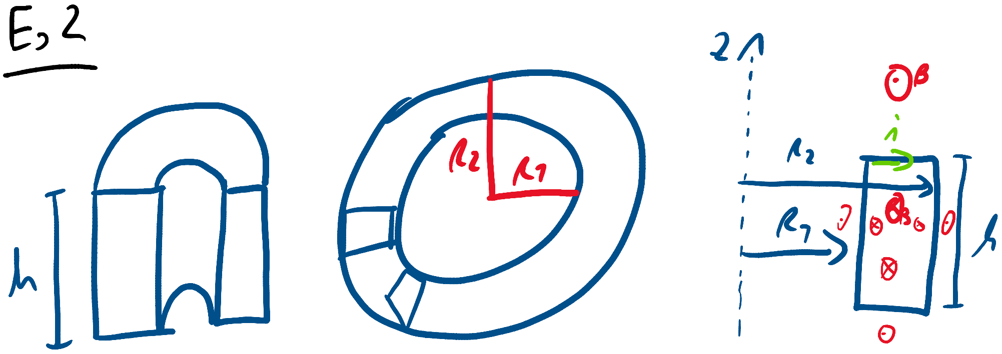
$$\Phi_S(E) = \frac{Q_{TOT}}{\varepsilon_0}$$

$\Rightarrow$

$$\Phi_S(E) = \frac{q}{\varepsilon_0}$$

$$\Phi_{\text{superficie nostra}} = \frac{1}{6}\frac{q}{\varepsilon_0}$$

Es 2

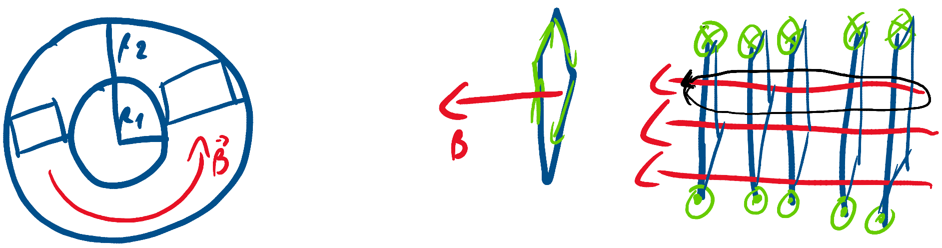


1) Ricavare l'espressione di B

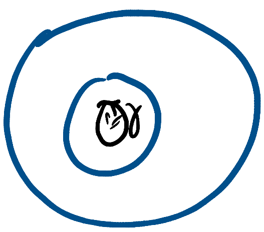
2) Ricavare l'induttanza

Sol

1) Usiamo Ampère



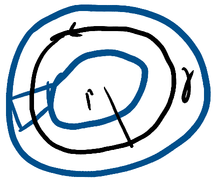
• Se $r < R_1$



$$\oint_\gamma B\,dl = \mu_0\, i_{conc}$$

$\Rightarrow$ $B = 0$ perché $i_{conc} = 0$

• Se $R_1 \le r \le R_2$

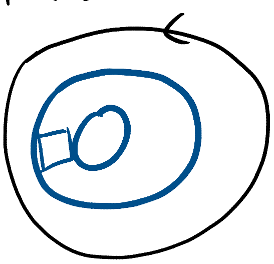


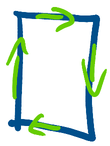
$$\oint_\gamma B\, dl = \mu_0\, i_{conc}$$

$$\Rightarrow B\, 2\pi r = \mu_0\, i \cdot N$$

$$\Rightarrow B = \frac{\mu_0\, i\, N}{2\pi r}$$

• Se $r > R_2$



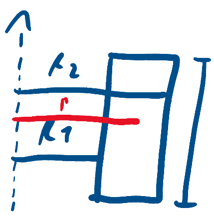


$$\oint_\gamma B\, dl = \mu_0\, i_{conc}$$

$\Rightarrow B = 0$ perché le i_{conc} si annullano a due a due

2)

$$L = \frac{\phi_S(B)}{i}$$



$S = h \cdot h$

$\Rightarrow dS = h \cdot dh$

$\Rightarrow dS = h \cdot dr$

Troviamo il flusso

$$\phi_S(B) = \int_S B\, dS = \int_{R_1}^{R_2} B \cdot h\, dr =$$

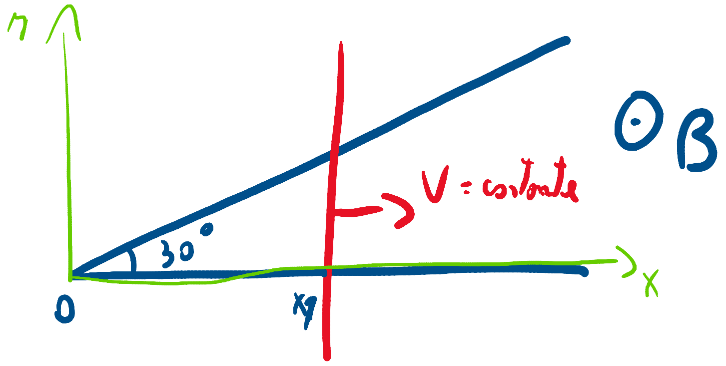
$$= \int_{R_1}^{R_2} \frac{\mu_0 N i}{2\pi r} h\, dr = \frac{\mu_0 N i}{2\pi} h \ln\left(\frac{R_2}{R_1}\right)$$

$$\Rightarrow L = \frac{\mu_0 N}{2\pi} \ln\left(\frac{R_2}{R_1}\right)$$

Appello 31/05/2023 (Autovalutazione)

giovedì 18 gennaio 2024 13:49

Es 2



$\theta = 30°$

$B = 1,2\ T$

$x_1 = 0,6\ m$

$\xi_{(x_1)} = 0,2\ V$

$R = 3,5\ \Omega$

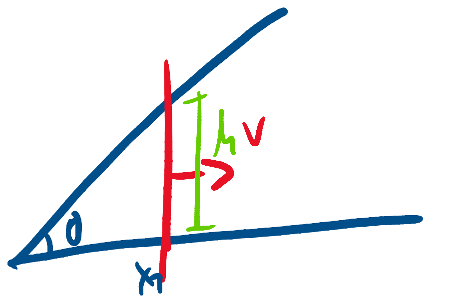
1) Trovare Velocità della sbarra

2) Trovare forza $F_{(x_1)}$

3) Calcolare il lavoro fatto da 0 a x_1

Sol

1)



$x(t) = x_0 + V \cdot t$

$\frac{h}{x(t)} = tg(\theta) \Rightarrow h = x\ tg(\theta)$

$\Rightarrow h = x(t)\ tg(\theta)$

Triangolo

$$\Phi_S(B) = \int_S B\, ds = B \cdot S_{figura} = B \cdot \frac{x(t) \cdot h}{2} =$$

$$= B\ x(t) \cdot x(t) \cdot tg(\theta) = \frac{B}{2} x(t)^2\ tg(\theta)$$

$\Rightarrow$

$$\frac{\partial \phi_s(\theta)}{\partial t} = B\,tg(\theta)\,2x(t)\cdot \dot{x} = B\,tg(\theta)\,x(t)\cdot V$$

$$\xi = -\frac{\partial \phi_s(\theta)}{\partial t} = -B\,tg(\theta)\cdot V\,x(t)$$

$$\xi(x_1) = 0{,}2$$

$\Rightarrow$

$$0{,}2 = -\underbrace{B\,tg(\theta)\cdot V\,x_1}_{<0}$$

$\Rightarrow$

$$V = \frac{0{,}2}{B\,tg(\theta)\,x_1}$$

2)

$$dF = i\,dl \wedge B$$

$\Rightarrow$

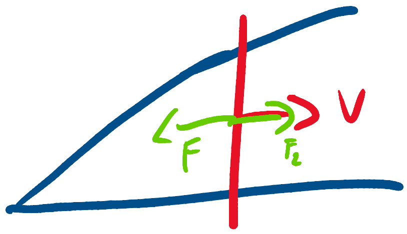
$$F = i\,B\,L$$

lunghezza sbarra

$\Rightarrow$

$$F = \frac{\xi}{R}\cdot B\,x(t)\cdot tg(\theta)$$

$$\Rightarrow F_{(x_1)} = \frac{\xi}{R} \cdot B \cdot tg(\theta) \cdot x_1$$



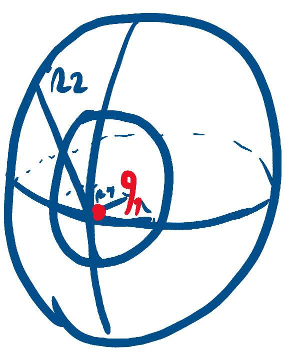
3)

$$F_2 = -F \qquad \Rightarrow \qquad |F_2| = |F|$$

$$\Rightarrow L = \int_\gamma F \, dl = \int_0^{x_1} \frac{\xi}{R} \cdot B \cdot tg(\theta) \, x \, dx =$$

$$= \frac{\xi}{R} B \, tg(\theta) \, \frac{x_1^2}{2}$$

<u>Es 3</u>



$R_1 = 10 \, cm$

$R_2 = 20 \, cm$

$q_1 = 3 \cdot 10^{5} C$

Trovare campo elettrico per

$\rho < R_1$

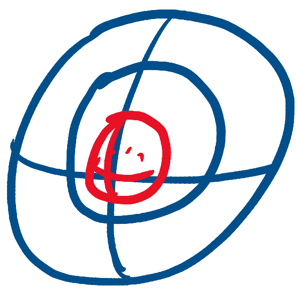
$R_1 \leq \rho \leq R_2$

$\rho > R_2$

Trovare campo elettrico quando una carica q_2 viene portata dall'infinito al conduttore

SOL

1) • $\rho < R_1$

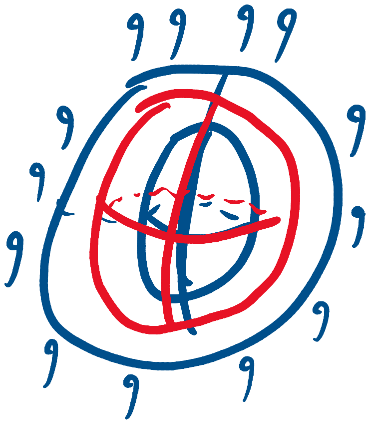


GAUSS

$$\oint_S E\,dS = \frac{Q_{int}}{\varepsilon_0} = \frac{q_1}{\varepsilon_0} \Rightarrow E \cdot 4\pi\rho^2 = \frac{q_1}{\varepsilon_0}$$

$$\Rightarrow E = \frac{q_1}{4\pi\varepsilon_0\rho^2}$$

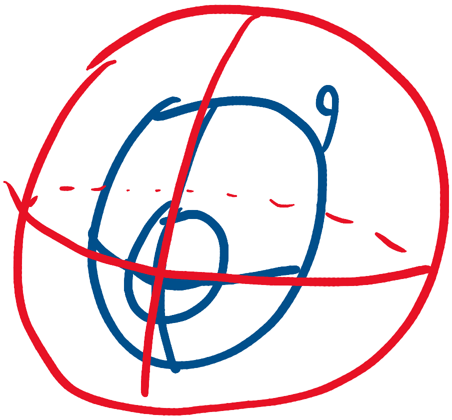
• $R_1 \leq \rho \leq R_2$



$Q_{int} = 0$ perché la sfera è un conduttore

$\Rightarrow \quad E = 0$

- $r > R_2$



$$\int_S E \, ds = \frac{Q_{int}}{\varepsilon_0} \quad \Rightarrow \quad E \, 4\pi r^2 = \frac{Q_{int}}{\varepsilon_0} = \frac{Q_{TOT}}{\varepsilon_0} = \frac{q_1}{\varepsilon_0}$$

$$\Rightarrow \quad E = \frac{q_1}{4\pi \varepsilon_0 r^2}$$

Per trovare il potenziale sfruttiamo il fatto che

$$V_{(a)} - V_{(b)} = \int_a^b E\, dl$$

$$\Rightarrow \quad V_{(r)} - V_{(\infty)} = \int_r^{+\infty} E\, dr = \int_r^{R_1} E\, dr + \int_{R_1}^{R_2} E\, dr + \int_{R_2}^{+\infty} E\, dr =$$

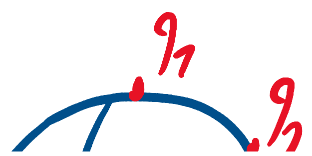
$$= \int_r^{R_1} \frac{q_1}{4\pi\varepsilon_0 r^2}\, dr + 0 + \int_{R_2}^{+\infty} \frac{q_1}{4\pi\varepsilon_0 r^2}\, dr =$$

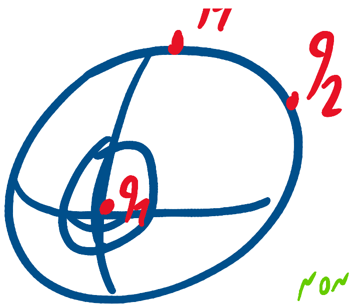
$$= \frac{q_1}{4\pi\varepsilon_0}\left(-\frac{1}{r}\right)\Bigg|_r^{R_1} + \frac{q_1}{4\pi\varepsilon_0}\left(-\frac{1}{r}\right)\Bigg|_{R_2}^{+\infty} =$$

$$= \frac{q_1}{4\pi\varepsilon_0}\left(-\frac{1}{R_1} + \frac{1}{r}\right) + \frac{q_1}{4\pi\varepsilon_0}\left(\frac{1}{R_2}\right) =$$

$$= \frac{q_1}{4\pi\varepsilon_0}\left(\frac{1}{r} + \frac{1}{R_2} - \frac{1}{R_1}\right)$$

2)





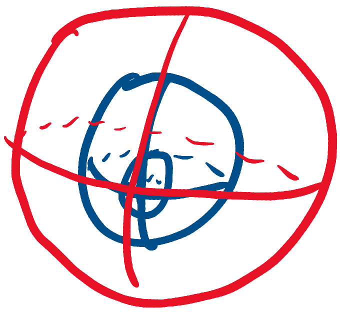
NON CAMBIA

$$r < R_1 \Rightarrow E = \frac{q_1}{4\pi\varepsilon_0 r^2}$$

NON C'É Q_{INT}

$$R_1 \le r \le R_2 \Rightarrow E = 0$$

$$r > R_2$$

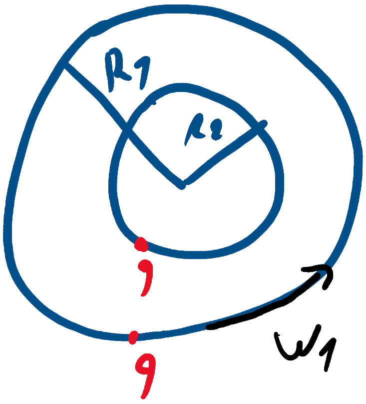


$$\oint_S E\,dS = \frac{Q_{INT}}{\varepsilon_0}$$

$$\Rightarrow E\,4\pi r^2 = \frac{Q_{INT}}{\varepsilon_0} = \frac{q_1 + q_2}{\varepsilon_0}$$

$$\Rightarrow E = \frac{q_1 + q_2}{4\pi\varepsilon_0 r^2}$$

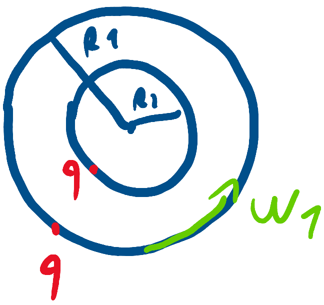
Es 1



1) Trovare B al centro dell'anello

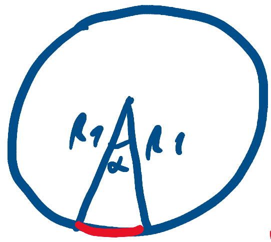
2) Trovare w_2, con $w_1 = 200$ rad/s, t.c. $B_{TOT} = 0$

Sol



$$dB = \frac{\mu_0 i_1}{4\pi} \cdot \frac{d\ell \wedge r}{r^3}$$

$$\Rightarrow \quad B = \int \frac{\mu_0 i}{4\pi} \frac{1}{R_1^2} \, dl$$



$dl = R_1 \, d\alpha$

$$\Rightarrow \quad B = \int_0^{2\pi} \frac{\mu_0 i}{4\pi} \frac{1}{R_1^2} \cdot R_1 \, d\alpha$$

$$\Rightarrow \quad B = \frac{\mu_0 i_1}{4\pi R_1} \cdot 2\pi = \frac{\mu_0 i_1}{2 R_1}$$

$$\Rightarrow \quad B = \frac{\mu_0 i_1}{2 R_1}$$

Troviamo i_1 :

$$i_1 = \frac{dQ}{dt} = \frac{\lambda \, dl}{dt} = \frac{\lambda \, R_1 \, d\alpha}{dt} = \lambda R_1 \omega$$

$$\lambda = \frac{dQ}{dl} \quad \text{oppure} \quad \lambda = \frac{Q}{l} = \frac{Q}{2\pi R_1}$$

$$\Rightarrow \quad i_1 = \frac{Q}{2\pi R_1} R_1 \omega = \frac{Q \omega_1}{2\pi}$$

$$B_{TOT} = 0 \quad \Rightarrow \quad B_1 + B_2 = 0$$

$$\Rightarrow$$

$$B_2 = -B_1$$

$$\Rightarrow$$

$$\frac{\mu_0 i_2}{2R_2} = -\frac{\mu_0 i_1}{2R_1}$$

$$\Rightarrow$$

$$\frac{\cancel{\mu_0}}{\cancel{2}R_2} \cdot \frac{\cancel{Q}\,w_2}{\cancel{2\pi}} = -\frac{\cancel{\mu_0}}{\cancel{2}R_1} \cdot \frac{\cancel{Q}\,w_1}{\cancel{2\pi}}$$

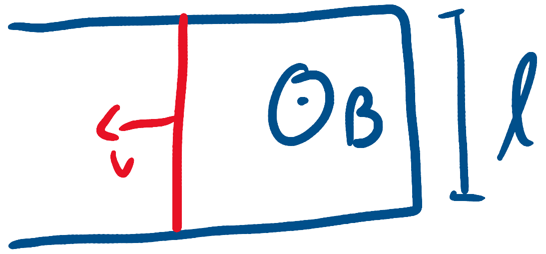
$$\Rightarrow \quad w_2 = -w_1 \frac{R_2}{R_1}$$

$\blacksquare$

.Appello 16/02/2023

venerdì 19 gennaio 2024 10:34

Es 1



$l = 10\ cm$

$V = 5\ m/s$

$B = 1{,}2\ T$

Determinare f.e.m, i_{ind} con $R = 475\ \Omega$, la Forza, la Potenza

Sol

$$\phi_S(B) = \int_S B\, ds = B \cdot \text{Superficie} = B\ x(t)\ l$$

$$= B(S_0 + v \cdot t)\ l$$

$\Rightarrow$

$$\frac{\partial \phi_S(B)}{\partial t} = B\, l\, V$$

$$\Rightarrow \xi = -B\, l\, V$$

$$\Rightarrow i_{ind} = -\frac{B\, l\, V}{R}$$

$$dF = i \, dl \wedge B$$

$$\Rightarrow F = i \, l \, B = -\frac{B^2 l^2 V}{R}$$

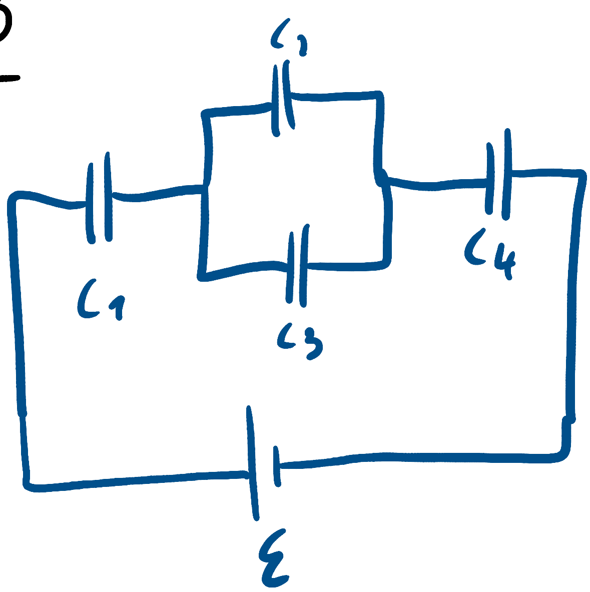
$$W = i^2 R = \frac{B^2 L^2 V^2}{R^2} \cdot R = \frac{B^2 L^2 V^2}{R}$$

$$W = \frac{\partial L}{\partial t}$$

$$L = \int_\gamma F \, \partial l = F \cdot x(t)$$

$$\frac{\partial L}{\partial t} = F \cdot V = \frac{B^2 L^2 V^2}{R}$$

Es 3



$C_1 = 10 \, \mu F$

$C_2 = 20 \, \mu F$

$C_3 = 30 \, \mu F$

$C_4 = 40 \, \mu F$

$\mathcal{E} = 12 \, V$

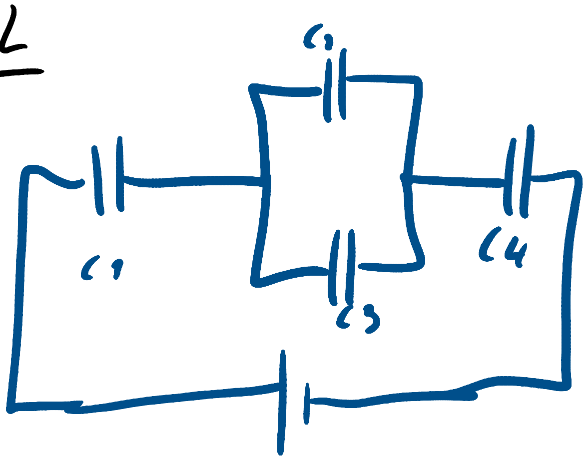
Trovare

1) Capacità Equivalente

2) Carica su C_2

3) ΔV in C_4

<u>Sol</u>



$$C_{eq_{23}} = C_2 + C_3$$

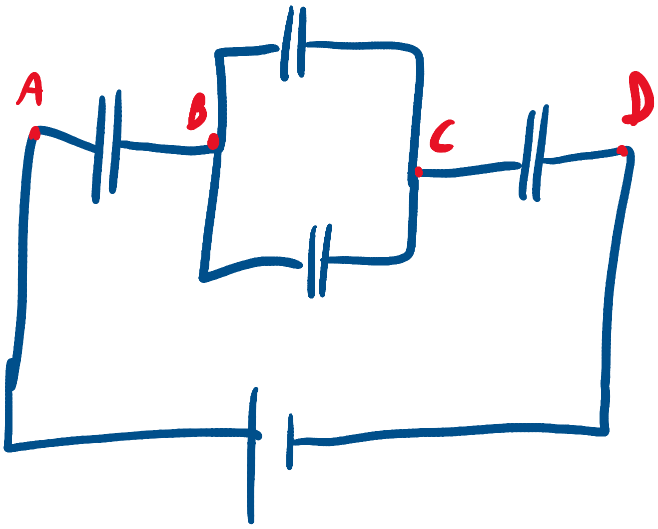
$$\frac{1}{C_{eq_{1234}}} = \frac{1}{C_1} + \frac{1}{C_{eq_{23}}} + \frac{1}{C_4}$$

$$\Rightarrow C_{eq_{1234}} = \left(\frac{1}{C_1} + \frac{1}{C_{eq_{23}}} + \frac{1}{C_4}\right)^{-1}$$

b)

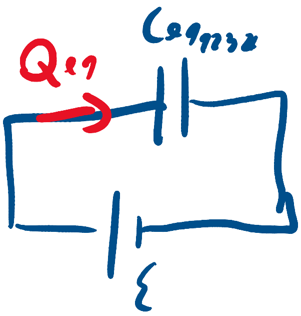
$C = \frac{Q}{\ }$

b)



$$C = \frac{Q}{\Delta V}$$

$$Q = C_{eq} \cdot \varepsilon$$



$$Q_2 = C_2 \cdot \Delta V_{(BC)}$$

$$\Delta V_{(AD)} = 12\,V = \varepsilon$$

$$\Delta V_{(AD)} = \Delta V_{AB} + \Delta V_{BC} + \Delta V_{CD} =$$

$$= \frac{Q_{eq}}{C_1} + \Delta V_{BC} + \frac{Q_{eq}}{C_4}$$

$$\Rightarrow \quad \frac{Q_{eq}}{C_1} + \frac{Q_{eq}}{C_4} + \Delta V_{(BC)} = \varepsilon$$

$$\Rightarrow \quad \Delta V_{(BC)} = \varepsilon - \frac{Q_{eq}}{C_1} - \frac{Q_{eq}}{C_4}$$

Dunque

$$Q_2 = \Delta V_{(BC)} \cdot C_2$$

$Q_2 = $ [illegible]

c)

$$\Delta V_{(CD)} = \frac{Q_{4,1}}{C_4}$$

E, 2

Determinare

a) Periodo di rotazione

b) il raggio dell'elica cilindrica

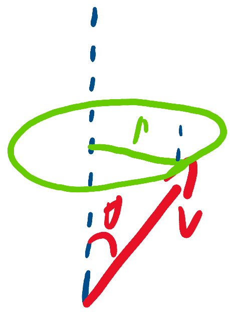
c) il passo

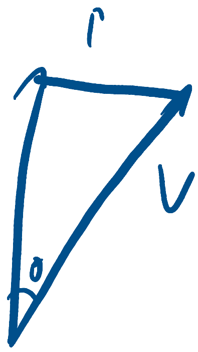
SOL

a) $\omega = \frac{2\pi}{T} \Rightarrow T = \frac{2\pi}{\omega}$

$$\omega = \frac{V}{r}$$

$$\Rightarrow \quad T = \frac{2\pi}{V} \cdot r$$





$$r = V \sin(\theta)$$

$$\Rightarrow$$

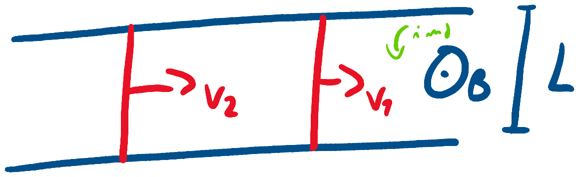
$$T = \frac{2\pi}{V} r = \frac{2\pi}{V} \cdot V \sin(\theta) = 2\pi \sin(\theta)$$

* Da finire *

Appello 28/06/2023

venerdì 19 gennaio 2024 15:12

Es 2



$L = 40\ cm$

$B = 1{,}2\ T$

$v_1 = 10\ m/s$

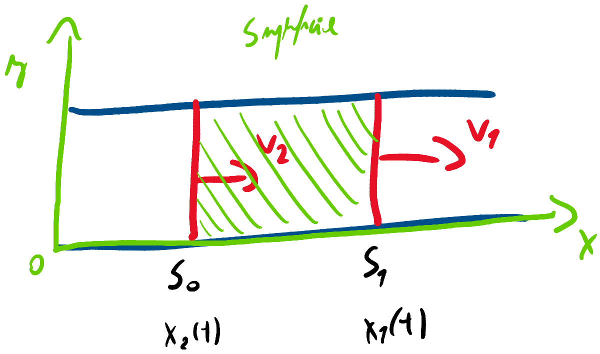
$v_2 = \frac{v_1}{2} = 5\ m/s$

$i_{ind} = 0{,}24\ A$

Calcolare

1) La resistenza in ciascuna barretta
2) La forza agente su ciascuna barretta
3) La carica che ha percorso il circuito dopo $t = 10\,s$

Sol



$$x_1(t) = (S_1 + v_1 t)$$

$$x_2(t) = (S_0 + v_2 \cdot t)$$

$$\text{Superficie} = (x_1(t) - x_2(t)) \cdot L$$

$$\phi_S(B) = \int_S B\, ds = B \cdot S = B \cdot L \cdot (x_1(t) - x_2(t)) =$$

$$= B \cdot L \cdot (S_1 + v_1 t - S_0 - v_2 t)$$

$$\Rightarrow \frac{\partial \phi_S(B)}{\partial t} = BLv_1 - BLv_2 = BL(v_1 - v_2)$$

$$\frac{d\Phi_S(v)}{dt} = BLv_1 - BLv_2 = BL(v_1 - v_2)$$

$$\Rightarrow \quad \xi = -(BLv_1 - BLv_2) = BLv_2 - BLv_1 = BL(v_2 - v_1)$$

$$\Rightarrow \quad i_{ind} = \frac{\xi}{R}$$ (gira in senso orario)

$$i_{ind} = 0{,}24$$

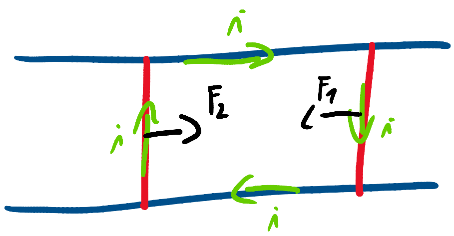
$$\Rightarrow \quad \frac{\xi}{R} = 0{,}24$$

$$\Rightarrow \quad R = \frac{\xi}{0{,}24} \quad \Rightarrow \quad R = \frac{BL(v_2 - v_1)}{0{,}24}$$

Ciascuna barra avrà

$$R_{NUOVA} = \frac{R}{2}$$

2)



$$dF_1 = i\, dl \wedge B = i_{ind}\, L\, B =$$

$$dF_2 = -dF_1$$

3)

$$i = \frac{dQ}{dt} \Rightarrow dQ = i\,dt$$

$$\Rightarrow Q = \int_0^{10} i\,dt = 10 \cdot i$$

Es 3

$R = 1\,m$

$\rho = 10^{-6}\,C/m^3$

$q = 1{,}6 \cdot 10^{-19}\,C$

$m = 10^{-19}\,kg$

$R_1 = R/2$

$R_2 = 4R$

Determinare

a) L'accelerazione nel punto R_1

b) L'accelerazione nel punto R_2

c) La velocità con cui la particella arriva nel punto R_2

SOL

a) Sappiamo che

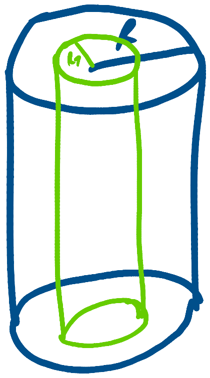
$$F = m \cdot a$$

e

$$F = E \cdot q$$

Troviamo pertanto la nostra E in R_1; usiamo GAUSS

Troviamo pertanto la nostra E in R_1; usiamo [illegible]



$$\Phi_S(E) = \frac{Q_{int}}{\varepsilon_0}$$

$$\Rightarrow \int_S E\, ds = \frac{Q_{int}}{\varepsilon_0}$$

$$\Rightarrow E \cdot S = \frac{Q_{int}}{\varepsilon_0}$$

$$\Rightarrow E \cdot 2\pi R_1 h = \frac{Q_{int}}{\varepsilon_0}$$

Volume del cilindro che contiene Q_{int}

(raggio tra 0 ed R_1)

$$\Rightarrow E = \frac{Q_{int}}{2\pi R_1 h\, \varepsilon_0} = \frac{\rho \cdot V}{2\pi R_1 h\, \varepsilon_0} =$$

$$= \frac{\rho \cdot \cancel{\pi} \cancel{h} R_1^{\cancel{2}}}{2 \cancel{\pi} \cancel{R_1} \cancel{h}\, \varepsilon_0} = \frac{\rho \cdot R_1}{2\varepsilon_0}$$

$$V = \pi r^2 \cdot h$$

$$\Rightarrow dV = \pi h\, 2r\, dr$$

$$\Rightarrow V = \int_0^{R_1} 2\pi h\, r\, dr =$$

$$= 2\pi h \left.\frac{r^2}{2}\right|_0^{R_1} =$$

$$= \pi h R_1^2$$

$$\Rightarrow E = \frac{\rho \cdot R_1}{2\varepsilon_0}$$

Dunque

$$F = E \cdot q = \frac{\rho R_1}{2\varepsilon_0} \cdot q$$

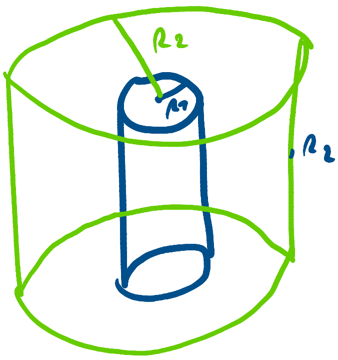
$$F = m \cdot a$$

$$\Rightarrow m \cdot a = \frac{\rho R_1 q}{2\varepsilon_0}$$

$$\Rightarrow a_{(R_1)} = \frac{\rho R_1 q}{2\varepsilon_0 m} = \frac{\rho \cdot q \cdot R}{4\varepsilon_0 m}$$

$$\Rightarrow a_{(R_1)} = \frac{\rho R q}{2\varepsilon_0 m} = \frac{\dots}{4\varepsilon_0 m}$$

b) Procediamo come prima



$$\oint_S(E) = \frac{Q_{int}}{\varepsilon_0}$$

$$\Rightarrow \int_S E\, dS = \frac{Q_{int}}{\varepsilon_0}$$

$$\Rightarrow E \cdot S = \frac{Q_{TOT}}{\varepsilon_0}$$

$$\Rightarrow E \cdot 2\pi R_2 h = \frac{Q_{TOT}}{\varepsilon_0}$$

volume che contiene Q_{TOT} (raggio tra 0 e R)

$$\Rightarrow E = \frac{Q_{TOT}}{2\pi R_2 h \varepsilon_0} = \frac{\rho \cdot V}{2\pi R_2 h \varepsilon_0} = \frac{\rho \cdot \cancel{\pi} R^2 \cancel{h}}{2\cancel{\pi} R_2 \cancel{h} \varepsilon_0} =$$

$$= \frac{\rho R^2}{2 R_2 \varepsilon_0} = \frac{\rho R^2}{2\varepsilon_0 \cdot 4R} = \frac{\rho R}{8\varepsilon_0}$$

$$\Rightarrow E = \frac{\rho R}{8\varepsilon_0}$$

Dunque

$$F = E \cdot q$$

$$F = m \cdot a$$

$$\Rightarrow \dots \Rightarrow a_{(R_2)} = \frac{\rho R q}{8\varepsilon_0 m}$$

c) Sfruttiamo il fatto che

$$\frac{1}{2} m V_f^2 - \frac{1}{2} m V_i^2 = L$$

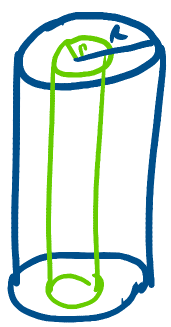
Adesso

$$L = \int_\gamma F\, dl = \int_\gamma E \cdot q\, dl = \int_{R_1}^{R_2} E q\, dl =$$

$$= \int_{R_1}^{R} E \cdot q\, dl + \int_{R}^{R_2} E \cdot q\, dl$$

Dobbiamo trovare allora delle espressioni generali per E quando $r < R$ e $r > R$:

• Se $r < R$



$$\phi_S(E) = \frac{Q_{int}}{\varepsilon_0}$$

$$\Rightarrow \int_S E\, ds = \frac{Q_{int}}{\varepsilon_0}$$

$$\Rightarrow E \cdot 2\pi r h = \frac{Q_{int}}{\varepsilon_0}$$

$$\Rightarrow E = \frac{Q_{int}}{2\pi r h \varepsilon_0} = \frac{\rho \cdot V}{2\pi r h \varepsilon_0} =$$

" raggio tra 0 ed r

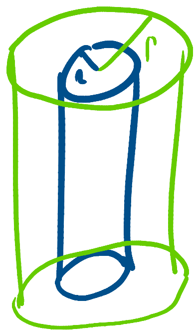
$$V = \pi r^2 h$$
$$\Rightarrow dV = \pi h\, 2r\, dr$$
$$\Rightarrow V = \int_0^r \pi h\, 2r\, dr$$
$$\Rightarrow V = 2\pi h \left.\frac{r^2}{2}\right|_0^r = \pi r^2 h$$

$$= \frac{\rho \cdot \cancel{\pi} r^{\cancel{2}} \cancel{h}}{2\cancel{\pi} \cancel{r} \cancel{h} \varepsilon_0} = \frac{\rho r}{2\varepsilon_0}$$

$$\Rightarrow E = \frac{\rho r}{\ldots}$$

$$\Rightarrow \quad E = \frac{\rho r}{2\varepsilon_0}$$

• Se $r > R$



$$\phi_S(E) = \frac{Q_{int}}{\varepsilon_0}$$

$$\Rightarrow \int_S E\, ds = \frac{Q_{int}}{\varepsilon_0}$$

$$\Rightarrow E \cdot S = \frac{Q_{int}}{\varepsilon_0}$$

$$\Rightarrow$$

$$E \cdot 2\pi r h = \frac{Q_{int}}{\varepsilon_0} = \frac{Q_{TOT}}{\varepsilon_0}$$

$$\Rightarrow$$

r raggio tra 0 ed R

$$E = \frac{Q_{TOT}}{2\pi r h \varepsilon_0} = \frac{\rho V}{2\pi r h \varepsilon_0} = \frac{\rho \cdot \cancel{\pi} R^2 \cancel{h}}{2 \cancel{\pi} r \cancel{h} \varepsilon_0} =$$

$$= \frac{\rho R^2}{2 r \varepsilon_0}$$

$$\Rightarrow E = \frac{\rho R^2}{2 r \varepsilon_0}$$

Troviamo L :

$$L = \int_{R_1}^{R} E_{(r<R)} \cdot q \, dl + \int_{R}^{R_2} E_{(r>R)} \cdot q \, dl =$$

$$= \int_{R_1}^{R} \frac{\rho r}{2\varepsilon_0} \cdot q \, dr + \int_{R}^{R_2} \frac{\rho R^2}{2 r \varepsilon_0} \cdot q \, dr =$$

$$\Big|^{R_2}$$

$$= \frac{\rho \cdot q}{2\varepsilon_0} \, \frac{r^2}{2} \Big|_{R_1}^{R} + \frac{\rho R^2}{2\varepsilon_0} q \cdot \ln(r) \Big|_{R}^{R_2} =$$

$$= \frac{\rho q}{2\varepsilon_0} \left(\frac{R^2}{2} - \frac{R_1^2}{2} \right) + \frac{\rho R^2}{2\varepsilon_0} q \cdot \ln\left(\frac{R_2}{R}\right)$$

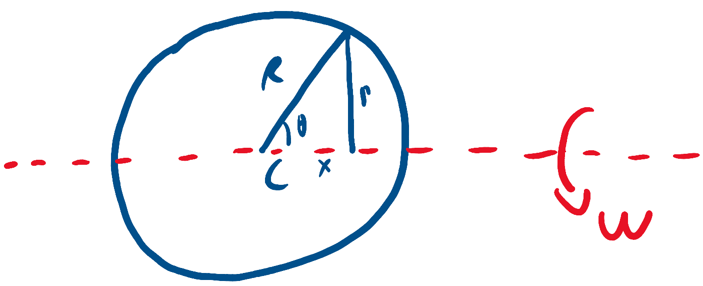
$\Rightarrow$

$$\frac{1}{2} m V_f^2 - \frac{1}{2} m \underbrace{V_i^2}_{=0} = L$$

$\Rightarrow$

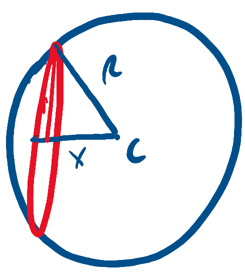
$$V_f = \sqrt{\frac{2L}{m}}$$

Es 1



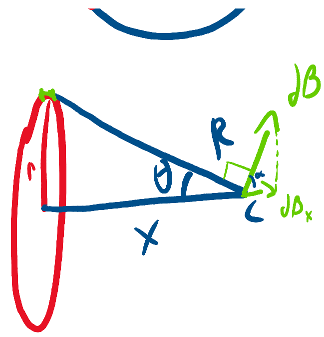
Calcolare B

SOL



dB

$r = R \sin(\theta)$



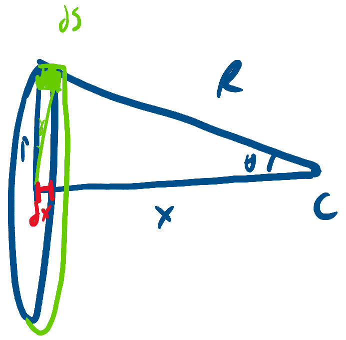
$r = R \sin(\theta)$

$dB_x = dB \cos(\alpha) = dB \cos\left(\frac{\pi}{2} - \theta\right) = dB \sin(\theta)$

$$dB_x = dB \sin(\theta) = \frac{\mu_0 i}{4\pi} \frac{dl \wedge R}{R^3} \sin(\theta) = \frac{\mu_0 i}{4\pi} \frac{\sin(\theta)}{R^2} dl$$

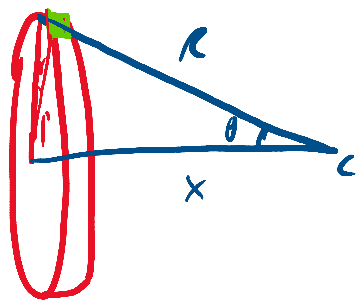
Capiamo chi è la corrente i:

$$i = \frac{dQ}{dt} = \frac{\sigma\, ds}{dt} = \frac{\sigma\, r\, d\gamma\, dx}{dt} = \sigma r \omega\, dx = \sigma r \omega R\, d\theta$$



$ds = r\, d\gamma\, dx$

$dx = R\, d\theta$



$$\Rightarrow \quad dB_x = \frac{\mu_0 \sigma r \omega R\, d\theta}{4\pi R^2} \sin(\theta)\, dl = \frac{\mu_0 \sigma r \omega \sin(\theta)\, d\theta}{4\pi R} dl$$

$$\Rightarrow \quad B = \frac{\mu_0 \sigma r \omega \sin(\theta)\, d\theta}{4\pi R} \cdot 2\pi r = \frac{\mu_0 \sigma \omega r^2 \sin(\theta)\, d\theta}{2R}$$

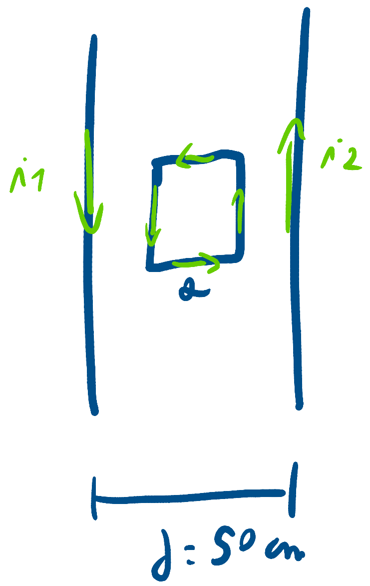
$$= \frac{\mu_0 \sigma \omega R^2 \sin^2(\theta) \sin(\theta)\, d\theta}{2R} = \frac{\mu_0 \omega \sigma R \sin^3(\theta)}{2} d\theta$$

$$B_{TOT} = \frac{\mu_0 \sigma R \omega}{2}\int_0^{\pi} \sin^3(\theta)\, d\theta = \dots$$

Appello 20/07/2023

sabato 20 gennaio 2024 10:34

Es 2

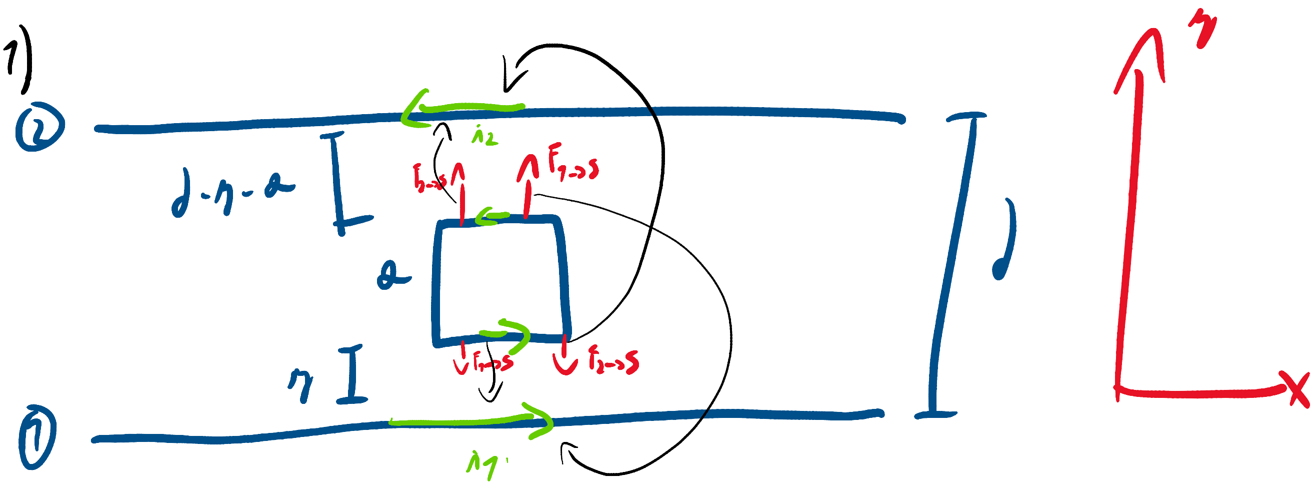


1) Determinare la posizione di equilibrio della spira nello spazio compreso fra i due fili

2) Determinare la natura dell'equilibrio

SOL

1)



$F_{TOT} = 0$

$$\Rightarrow \frac{\mu_0 i_2 i_s a}{2\pi(d-y-a)} + \frac{\mu_0 i_s i_1 a}{2\pi(y+a)} - \frac{\mu_0 i_s i_1 a}{2\pi y} - \frac{\mu_0 i_2 i_s a}{2\pi(d-y)} = 0$$

$$\Rightarrow \frac{i_2}{d-y-a} + \frac{i_1}{y+a} - \frac{i_1}{y} - \frac{i_2}{d-y} = 0$$

$$\Rightarrow i_2\left(\frac{1}{d-y-a} - \frac{1}{d-y}\right) + i_1\left(\frac{1}{y+a} - \frac{1}{y}\right) = 0$$

$$\Rightarrow i_2\left(\frac{d-y-d+y+a}{(d-y-a)(d-y)}\right) + i_1\left(\frac{y-y-a}{y(y+a)}\right) = 0$$

$$\Rightarrow i_2\left(\frac{a}{(d-y-a)(d-y)}\right) + i_1\left(\frac{-a}{y(y+a)}\right) = 0$$

$$\Rightarrow \frac{i_2 \cdot \cancel{a}}{(d-y-a)(d-y)} = \frac{i_1 \cancel{a}}{y(y+a)}$$

$$\Rightarrow i_2(y+a)y = i_1(d-y)(d-y-a)$$

$\Rightarrow$
$$i_2 y^2 + i_2 y a = i_1 (d^2 - dy - ad - yd + y^2 + ya)$$

$\Rightarrow$
$$i_2 y^2 + i_2 y a = i_1 d^2 - i_1 d y - i_1 a d - i_1 y d + i_1 y^2 + i_1 y a$$

$\Rightarrow$
$$i_2 y^2 + i_2 y a + i_1 d y + i_1 d y - i_1 y^2 - i_1 y a = i_1 d^2 - i_1 a d$$

$\Rightarrow$
$$y^2 (i_2 - i_1) + y (i_2 a + 2 i_1 d - i_1 a) - i_1 d^2 + i_1 a d = 0$$

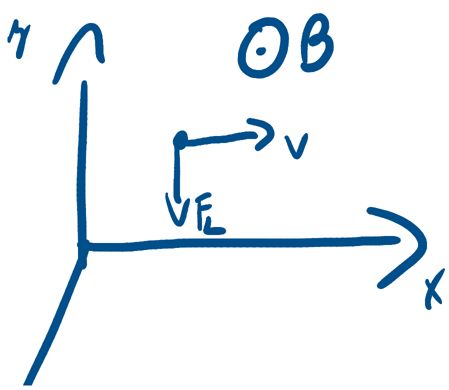
$\Rightarrow$ Ricavo y^*

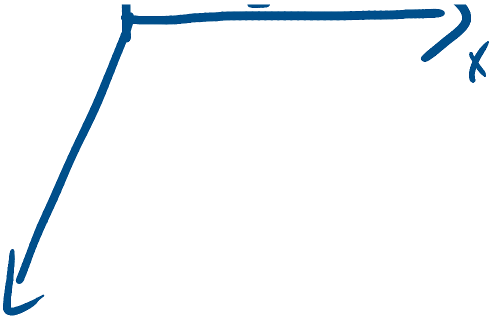
2) Derivo rispetto a y e trovo punti di max e min

Se $y = \max$ $\Rightarrow$ INSTABILE

$y = \min$ $\Rightarrow$ STABILE

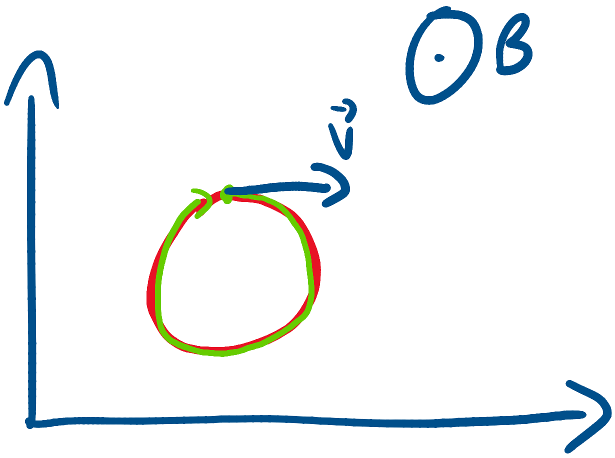
Es 3





Calcolare campo elettrico per avere una traiettoria rettilinea

SOL



Forza di Lorentz

$$F = q(E + v \wedge B) = 0$$

$\Rightarrow$

$$qE = -q(v \wedge B)$$

$$v = (v, 0, 0)$$

$$B = (0,0,B)$$

$$\Rightarrow v \wedge B = \begin{vmatrix} i & j & k \\ v & 0 & 0 \\ 0 & 0 & B \end{vmatrix} = (0, -vB, 0)$$

$\Rightarrow$

$$qE = -q(v \wedge B)$$

$\Rightarrow$

$$E = (0, vB, 0)$$

Adesso

$$F_{ele} = E \cdot q$$

$$F = m \cdot a = m\, \omega^2 R$$

Forza centripeta

$\Rightarrow$

$$E \cdot q = m\, \omega^2 R$$

$\Rightarrow$

$$vBq = m \frac{v^2}{R^2} R$$

$\Rightarrow$

$$vBq = m \frac{v^2}{R}$$

$$\Rightarrow \quad VBq = m\frac{V^2}{R}$$

$$\Rightarrow \quad Bq = \frac{m}{R}V$$

$$\Rightarrow \quad V = \frac{BqR}{m}$$

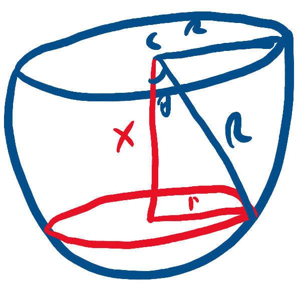
$$\Rightarrow \quad E = VB = \frac{B^2qR}{m}$$

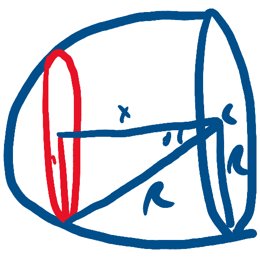
$\blacksquare$

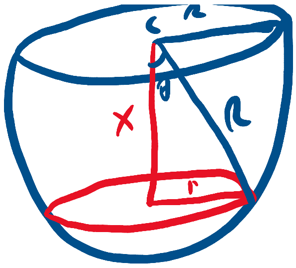
Es 7

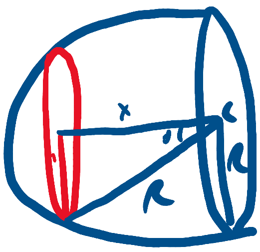
Trovare campo elettrico

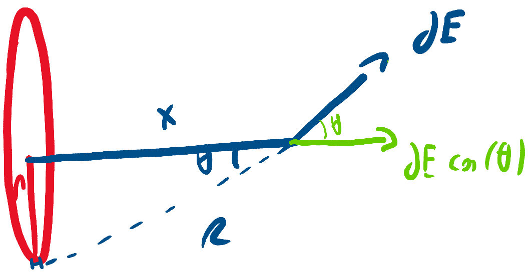
Sol











$$x = R\cos(\theta) \Rightarrow \cos(\theta) = \frac{x}{R}$$

$$r = R\sin(\theta)$$

$$dE_x = dE\cos(\theta) = \frac{1}{4\pi\varepsilon_0}\frac{dQ}{R^2}\cos(\theta) = \frac{\lambda\, dl}{4\pi\varepsilon_0 R^2}\cos(\theta)$$

$\Rightarrow$

$$E_x = \frac{\lambda}{4\pi\varepsilon_0 R^2}\cdot 2\pi r\cdot\cos(\theta) =$$

$$= \frac{\lambda\, 2\pi r}{4\pi\varepsilon_0 R^2}\cdot\frac{x}{R} = \frac{\lambda\, 2\pi r\, x}{4\pi\varepsilon_0 R^3}$$

Questa E_x è il contributo di un filo, a noi serve in tutta la mezza sfera.
Per fare ciò consideriamo tutta carica Q che è sul filo come un dQ di tutta la superficie:

(Per il filo) $\lambda = \frac{Q}{l} \Rightarrow Q = \lambda l = \lambda\cdot 2\pi r$

(Per il filo) $\lambda = \frac{Q}{l} \Rightarrow Q = \lambda l = \underbrace{\lambda \cdot 2\pi r}$

Ecco perchè non lo abbiamo semplificato

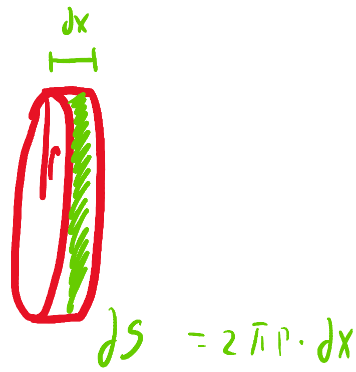
Dunque

$$dE_{TOT} = \frac{dQ \cdot x}{4\pi \varepsilon_0 R^3} = \frac{\sigma \, dS \, x}{4\pi \varepsilon_0 R^3} =$$

$$= \frac{\sigma \, dS \, R \cos(\theta)}{4\pi \varepsilon_0 R^3} = \frac{\sigma \cos(\theta) \, dS}{4\pi \varepsilon_0 R^2} =$$

$$= \frac{\frac{Q}{2\pi R^2} \cdot \cos(\theta) \, dS}{4\pi \varepsilon_0 R^2} = \frac{Q \cos(\theta) \, dS}{8 \varepsilon_0 \pi^2 R^4} =$$

$$= \frac{Q \cos(\theta)}{8 \varepsilon_0 \pi^2 R^4} 2\pi r \, dx =$$



$$= \frac{Q \cos(\theta) \, r \, dx}{4\pi \varepsilon_0 R^4} =$$

$$= \frac{Q \cos(\theta) \cdot R \sin(\theta)}{} dx =$$

$$= \frac{Q \cos(\theta) \cdot R \sin(\theta)}{4\pi \varepsilon_0 R^4} \, dx =$$

$$= \frac{Q \cos(\theta) \sin(\theta)}{4\pi \varepsilon_0 R^3} \, dx = \frac{Q \cos(\theta) \sin(\theta)}{4\pi \varepsilon_0 R^3} \, R \, d\theta =$$

$$= \frac{Q}{4\pi \varepsilon_0 R^2} \sin(\theta) \cos(\theta) \, d\theta$$

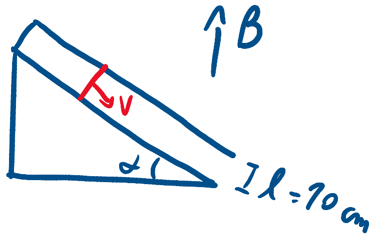
$$\Rightarrow$$

$$E_{TOT} = \int_0^{\pi/2} \frac{Q}{4\pi \varepsilon_0 R^2} \sin(\theta) \cos(\theta) \, d\theta = \ldots =$$

Appello 15/09/2023

sabato 20 gennaio 2024 16:47

Es 1

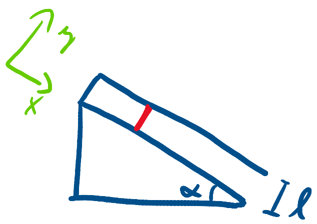


Calcolare

1) f.e.m e i_{ind}

2) La velocità limite

Sol



$x(t) = S_0 + V_0 \cdot t + \frac{1}{2} a t^2$

$V(t) = V_0 + a t$

$B \Rightarrow B \cos(\alpha)$

$$\phi_S(B) = \int_S B \cos(\alpha)\, dS = B \cos(\alpha) \cdot S_{superficie} =$$

$$= B \cos(\alpha) \cdot l \cdot x(t) =$$

$$= B\, l \cos(\alpha) \left(S_0 + V_0 t + \frac{1}{2} a t^2\right) =$$

$$= B l \cos(\alpha) \cdot \frac{1}{2} a t^2$$

$$\frac{d\phi_S(B)}{dT} = B\, l \cos(\alpha) \cdot a t$$

$$\xi = - B l \cos(\alpha) \cdot a t = - B l \cos(\alpha)\, V(t)$$

$$\Rightarrow i_{ind} = \frac{\xi}{R} = - \frac{B l \cos(\alpha)}{R} V(t)$$

2)

$$F = m \cdot a$$

$$F = i \, l \, B \cos(\alpha)$$

$$F_{p_{\parallel}} = m \cdot g \sin(\alpha)$$

IMPONGO

$$\Rightarrow \quad m \cdot a = i \, l \, B \cos(\alpha) + m \, g \sin(\alpha) = 0$$

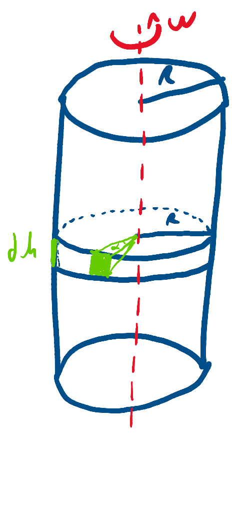
$$\Rightarrow \quad m \, g \sin(\alpha) = \frac{B^2 l^2 \cos^2(\alpha) \, v(t)}{R}$$

$$\Rightarrow \quad V = \frac{m \, g \sin(\alpha) \cdot R}{B^2 l^2 \cos^2(\alpha)}$$

Es 2

Determinare B

SoL



$$i = \frac{dQ}{dt} = \frac{\sigma \, ds}{dt}$$

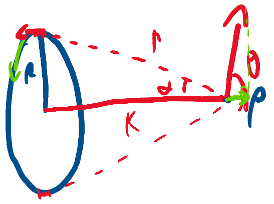
$$ds = R \, d\alpha \, dh$$

$$\Rightarrow \quad i = \frac{\sigma R \, d\alpha \, dh}{dt} = \sigma R \, \omega \, dh$$

$$dB = \frac{\mu_0 i}{4\pi} \frac{dl \wedge \bar{r}}{r^3}$$

$$\Rightarrow \quad dB = \frac{\mu_0 i}{4\pi} \frac{dl}{R^2}$$

Consideriamo un filo circolare e poi passiamo a tutto il cilindro



$dB \cos(\theta)$

$\cos(\theta) = [\cos(\frac{\pi}{2} - \alpha)] = \sin(\alpha) = R/r$

$$r = \sqrt{R^2 + K^2}$$

$$\frac{K}{r} = \cos(\theta) \Rightarrow \cos(\theta) = \frac{R}{\sqrt{R^2+K^2}}$$

$$dB \cos(\theta) = \frac{\mu_0 i}{4\pi} \frac{dl}{r^2} \cos(\theta)$$

$$\Rightarrow \quad dB \cos(\theta) = \frac{\mu_0 i}{4\pi} \frac{1}{(K^2+R^2)} \cdot \frac{R}{\sqrt{K^2+R^2}} \cdot dl =$$

$$= \frac{\mu_0 i R}{4\pi (K^2+R^2)^{3/2}} dl$$

$$\Rightarrow \quad B \cos(\theta) = B_y = \frac{\mu_0 i R}{4\pi (K^2+R^2)^{3/2}} \cdot 2\pi R = \frac{\mu_0 i R^2}{2 (K^2+R^2)^{3/2}}$$

Dunque

$$B_{y_{\text{cilindrino}}} = \frac{\mu_0 i R^2}{2 (K^2+R^2)^{3/2}} = \frac{\mu_0 \sigma R w \, dh \; R^2}{2 (K^2+R^2)^{3/2}} =$$

$$= \frac{\mu_0 \sigma w R^3 \, dh}{2 (K^2+R^2)^{3/2}}$$

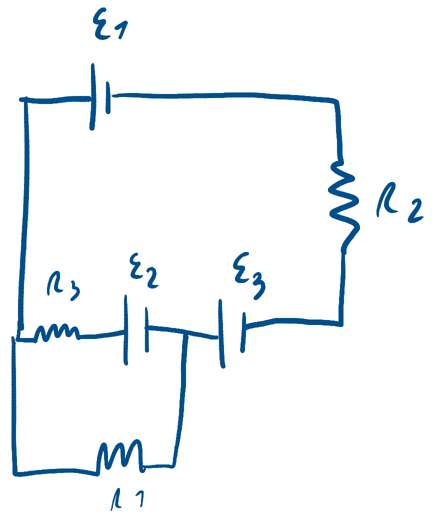
$$\Rightarrow \quad B_{y_{\text{CILINDRO}}} = \int \frac{\mu_0 \sigma w R^3}{2 (K^2+R^2)^{3/2}} dh =$$

$$\Rightarrow B_{y_{CILINDRO}} = \int \frac{\mu_0 \sigma w R^3}{2(K^2+R^2)^{3/2}} \, dh =$$

$$= \frac{\mu_0 \sigma w R^3}{2} \int_{-\infty}^{\infty} \frac{1}{(K^2+R^2)^{3/2}} \cdot dK = $$

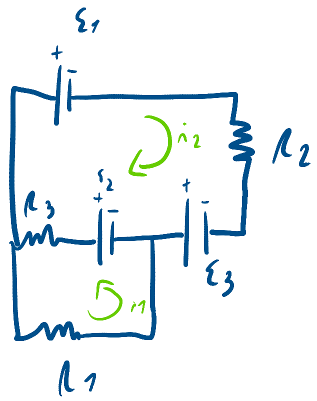
$K = R \, tg(u)$

Es 3



Trovare i_1 e i_2 e le potenze in R_1, R_2, R_3

Sol



$$\begin{cases} \mathcal{E}_3 + \mathcal{E}_2 - R_3(i_2 + i_1) - \mathcal{E}_1 - R_2 i_2 = 0 \\ \mathcal{E}_2 - R_3(i_1 + i_2) - R_1 i_1 = 0 \end{cases}$$

$$\Rightarrow \dots \Rightarrow \begin{cases} i_1 \\ i_2 \end{cases}$$

Troviamo le potenze

$$W_{(R_1)} = i_1^2 \cdot R_1$$

$$W_{(R_2)} = i_2^2 \cdot R_2$$

$$W_{(R_3)} = (i_1 + i_2)^2 \cdot R_3$$